Section 3: Implicit differentiation

Solutions to Exercise level 2

1. (i) Using the product rule: $\frac{d}{dx}(xy^2) = 1 \times y^2 + x \times 2y\frac{dy}{dx}$

$= y^2 + 2xy\frac{dy}{dx}$

(ii) Using the product rule: $\frac{d}{dx}(x\sin y) = 1 \times \sin y + x \times \cos y\frac{dy}{dx}$

$= \sin y + x\cos y\frac{dy}{dx}$

(iii) Using the quotient rule: $\frac{d}{dx}\left(\frac{y}{1+x}\right) = \frac{(1+x)\frac{dy}{dx} - 1 \times y}{(1+x)^2}$

$= \frac{1}{(1+x)}\frac{dy}{dx} - \frac{y}{(1+x)^2}$

2. $x^3 + y^3 - 3x - 6 = 0$

Differentiating implicitly: $3x^2 + 3y^2\frac{dy}{dx} - 3 = 0$

$3y^2\frac{dy}{dx} = 3 - 3x^2$

$\frac{dy}{dx} = \frac{1-x^2}{y^2}$

At turning points, $\frac{1-x^2}{y^2} = 0$

$1 - x^2 = 0$

$x = \pm 1$

When $x = 1$, $1 + y^3 - 3 - 6 = 0 \Rightarrow y^3 = 8 \Rightarrow y = 2$

When $x = -1$, $-1 + y^3 + 3 - 6 = 0 \Rightarrow y^3 = 4 \Rightarrow y = \sqrt[3]{4}$

so turning points are $(1, 2)$ and $(-1, \sqrt[3]{4})$

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3. $y^2 = \dfrac{x^2}{1+2x}$

Differentiating implicitly: $2y\dfrac{dy}{dx} = \dfrac{2x(1+2x) - x^2 \times 2}{(1+2x)^2} = \dfrac{2x+2x^2}{(1+2x)^2}$

$y\dfrac{dy}{dx} = \dfrac{x+x^2}{(1+2x)^2}$

$\dfrac{dy}{dx} = \dfrac{x(1+x)}{y(1+2x)^2}$

Using quotient rule

$y = \dfrac{x}{\sqrt{1+2x}}$

$\dfrac{dy}{dx} = \dfrac{x(1+x)}{\frac{x}{\sqrt{1+2x}}(1+2x)^2} = \dfrac{(1+x)}{(1+2x)^{\frac{3}{2}}}$

4. $y = 2^x$

$\ln y = \ln 2^x = x\ln 2$

Differentiating implicitly: $\dfrac{1}{y}\dfrac{dy}{dx} = \ln 2$

$\dfrac{dy}{dx} = y\ln 2 = 2^x \ln 2$