Section 3: The product and quotient rules

Exercise level 3 solutions

1. (i) $f(x) = (x-a)^2 g(x)$

$u = (x-a)^2 \quad \Rightarrow \frac{du}{dx} = 2(x-a)$

$v = g(x) \quad \Rightarrow \frac{dv}{dx} = g'(x)$

Using the product rule, $f'(x) = 2(x-a)g(x) + (x-a)^2 g'(x)$

$f'(a) = 0 + 0 = 0$

(ii) $f'(x) = 2(x-a)g(x) + (x-a)^2 g'(x)$

$f''(x) = 2g(x) + 2(x-a)g'(x) + 2(x-a)g'(x) + (x-a)^2 g''(x)$

$= 2g(x) + 4(x-a)g'(x) + (x-a)^2 g''(x)$

When $x = a$, $f''(x) = 2g(a) + 0 + 0 = 2g(a)$

Since $g(a) > 0$, $f''(a) > 0$

and so the stationary point is a minimum point.

2. (i) $y = \frac{x+1}{x-1}$

$u = x+1 \quad \Rightarrow \frac{du}{dx} = 1$

$v = x-1 \quad \Rightarrow \frac{dv}{dx} = 1$

$\frac{dy}{dx} = \frac{(x-1)-(x+1)}{(x-1)^2} = -\frac{2}{(x-1)^2}$

(ii) $y = \frac{x+1}{x-1} = \frac{x-1+2}{x-1}$

$= 1 + \frac{2}{x-1}$

$= 1 + 2(x-1)^{-1}$

$\frac{dy}{dx} = 0 + 2 \times -(x-1)^{-2} = -2(x-1)^{-2}$

$= -\frac{2}{(x-1)^2}$

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3. (i) $y = \frac{1}{g(x)} = [g(x)]^{-1}$

$$\frac{dy}{dx} = -[g(x)]^{-2} \times g'(x)$$

$$= -\frac{g'(x)}{[g(x)]^2}$$

(ii) $y = \frac{f(x)}{g(x)} = f(x) \times \frac{1}{g(x)}$

$$\frac{dy}{dx} = f'(x) \times \frac{1}{g(x)} + f(x) \times \frac{d}{dx}\left(\frac{1}{g(x)}\right)$$

$$= \frac{f'(x)}{g(x)} + f(x) \times -\frac{g'(x)}{[g(x)]^2}$$

$$= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{[g(x)]^2}$$

$$= \frac{f'(x)g(x)}{[g(x)]^2} - \frac{f(x)g'(x)}{[g(x)]^2}$$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

which is the quotient rule.