

Section 3: The product and quotient rules

Solutions to Exercise level 2

1. (i) $y = x(2x-1)^2$

Let $u = x \Rightarrow \frac{du}{dx} = 1$

Let $v = (2x-1)^2 \Rightarrow \frac{dv}{dx} = 2(2x-1) \times 2 = 4(2x-1)$

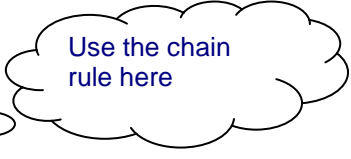
using the product rule: $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

$$= x \times 4(2x-1) + 1 \times (2x-1)^2$$

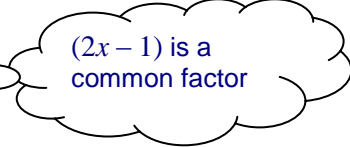
$$= 4x(2x-1) + (2x-1)^2$$

$$= (2x-1)(4x + (2x-1))$$

$$= (2x-1)(6x-1)$$



Use the chain rule here



$(2x-1)$ is a common factor

(ii) $y = x(2x-1)^2 = x(4x^2 - 4x + 1) = 4x^3 - 4x^2 + x$

$$\frac{dy}{dx} = 12x^2 - 8x + 1$$

(iii) $(2x-1)(6x-1) = 12x^2 - 8x + 1$

so the two answers are algebraically equivalent.

2. (i) $y = \frac{5x^2}{\sqrt{x+3}}$

Let $u = 5x^2 \Rightarrow \frac{du}{dx} = 10x$

Let $v = (x+3)^{\frac{1}{2}} \Rightarrow \frac{dv}{dx} = \frac{1}{2}(x+3)^{-\frac{1}{2}}$

using the quotient rule: $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{(x+3)^{\frac{1}{2}} \times 10x - 5x^2 \times \frac{1}{2}(x+3)^{-\frac{1}{2}}}{(x+3)}$$

$$= \frac{(x+3) \times 20x - 5x^2}{2(x+3)^{\frac{3}{2}}}$$

$$= \frac{15x^2 + 60x}{2(x+3)^{\frac{3}{2}}}$$

$$= \frac{15x(x+4)}{2(x+3)^{\frac{3}{2}}}$$

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$$(ii) \quad y = 5x^2(x+3)^{-\frac{1}{2}}$$

$$\text{Let } u = 5x^2 \Rightarrow \frac{du}{dx} = 10x$$

$$\text{Let } v = (x+3)^{-\frac{1}{2}} \Rightarrow \frac{dv}{dx} = -\frac{1}{2}(x+3)^{-\frac{3}{2}}$$

$$\text{using the product rule: } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{dy}{dx} = 5x^2 \times -\frac{1}{2}(x+3)^{-\frac{3}{2}} + (x+3)^{-\frac{1}{2}} \times 10x$$

$$= \frac{5x^2 \times -\frac{1}{2} + (x+3) \times 10x}{(x+3)^{\frac{3}{2}}}$$

$$= \frac{-5x^2 + (x+3) \times 20x}{2(x+3)^{\frac{3}{2}}}$$

$$= \frac{15x^2 + 60x}{2(x+3)^{\frac{3}{2}}}$$

$$= \frac{15x(x+4)}{2(x+3)^{\frac{3}{2}}}$$

(iii) Shown above

(iv) For $x = -3$ the denominator is zero so the function is undefined. For $x < -3$ the square root of a negative number is required so the function is undefined.

$$3. \quad y = x\sqrt{2x+5} \quad \text{for } x \geq -2.5$$

$$\text{Let } u = x \Rightarrow \frac{du}{dx} = 1$$

$$\text{Let } v = (2x+5)^{\frac{1}{2}} \Rightarrow \frac{dv}{dx} = \frac{1}{2}(2x+5)^{-\frac{1}{2}} \times 2$$

$$\text{using the product rule: } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{dy}{dx} = x \times \frac{1}{2}(2x+5)^{-\frac{1}{2}} \times 2 + 1 \times (2x+5)^{\frac{1}{2}}$$

$$= \frac{x + 2x + 5}{(2x+5)^{\frac{1}{2}}}$$

$$= \frac{3x+5}{\sqrt{2x+5}}$$

So $a = 3$ and $b = 5$

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4. $y = \frac{x}{\sqrt{3x-2}}$ for $x > \frac{2}{3}$

Let $u = x \Rightarrow \frac{du}{dx} = 1$

Let $v = (3x-2)^{\frac{1}{2}} \Rightarrow \frac{dv}{dx} = \frac{1}{2}(3x-2)^{-\frac{1}{2}} \times 3$

using the quotient rule: $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(3x-2)^{\frac{1}{2}} \times 1 - x \times \frac{1}{2}(3x-2)^{-\frac{1}{2}} \times 3}{3x-2} \\ &= \frac{(3x-2) - x \times \frac{3}{2}}{(3x-2)^{\frac{3}{2}}} \\ &= \frac{\frac{3}{2}x - 2}{(3x-2)^{\frac{3}{2}}} \end{aligned}$$

So $a = \frac{3}{2}$ and $b = -2$

5. $y = x\sqrt{1-x} = x(1-x)^{\frac{1}{2}}$

Let $u = x \Rightarrow \frac{du}{dx} = 1$

Let $v = (1-x)^{\frac{1}{2}} \Rightarrow \frac{dv}{dx} = \frac{1}{2}(1-x)^{-\frac{1}{2}} \times -1 = -\frac{1}{2\sqrt{1-x}}$

using the product rule: $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

$$\begin{aligned} &= x \times -\frac{1}{2\sqrt{1-x}} + \sqrt{1-x} \times 1 \\ &= -\frac{x}{2\sqrt{1-x}} + \sqrt{1-x} \end{aligned}$$

At turning point, $\frac{x}{2\sqrt{1-x}} = \sqrt{1-x}$

$$x = 2(1-x)$$

$$x = 2 - 2x$$

$$3x = 2$$

$$x = \frac{2}{3}$$

When $x = \frac{2}{3}$, $y = \frac{2}{3}(1 - \frac{2}{3})^{\frac{1}{2}} = \frac{2}{3\sqrt{3}}$

Turning point is $(\frac{2}{3}, \frac{2}{3\sqrt{3}})$.

Use the chain rule here

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$$6. y = \frac{x}{2x-1}$$

$$\text{Let } u = x \Rightarrow \frac{du}{dx} = 1$$

$$\text{Let } v = 2x-1 \Rightarrow \frac{dv}{dx} = 2$$

$$\begin{aligned} \text{Using the quotient rule, } \frac{dy}{dx} &= \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \\ &= \frac{1(2x-1) - 2x}{(2x-1)^2} \\ &= -\frac{1}{(2x-1)^2} \end{aligned}$$

Since $(2x-1)^2$ is always positive, $-\frac{1}{(2x-1)^2}$ is always negative for $x \neq \frac{1}{2}$,
and so the gradient of the curve is always negative for $x \neq \frac{1}{2}$.

$$7. y = x^2 \sqrt{2x-1}, \quad x > \frac{1}{2}, \text{ at the point where } x = 1$$

$$\text{Let } u = x^2 \Rightarrow \frac{du}{dx} = 2x$$

$$\text{Let } v = (2x-1)^{\frac{1}{2}} \Rightarrow \frac{dv}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2 = (2x-1)^{-\frac{1}{2}}$$

$$\text{Using the product rule: } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\begin{aligned} \frac{dy}{dx} &= x^2(2x-1)^{-\frac{1}{2}} + 2x(2x-1)^{\frac{1}{2}} \\ &= 1 \times 1 + 2 \times 1 \\ &= 3 \end{aligned}$$

$$\text{Gradient} = 3, \quad x = 1, \quad y = 1$$

$$\text{Equation of tangent: } y - 1 = 3(x - 1)$$

$$y = 3x - 2$$

$$8. y = \frac{x^2}{x-1}$$

$$\text{Let } u = x^2 \Rightarrow \frac{du}{dx} = 2x$$

$$\text{Let } v = x-1 \Rightarrow \frac{dv}{dx} = 1$$

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using the quotient rule, $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$$= \frac{2x(x-1) - x^2}{(x-1)^2}$$

$$= \frac{x(x-2)}{(x-1)^2}$$

At turning points, $\frac{x(x-2)}{(x-1)^2} = 0$

$$x = 0 \text{ or } x = 2$$

When $x = 0$, $y = 0$

When $x = 2$, $y = \frac{2^2}{2-1} = 4$

The turning points are $(0, 0)$ and $(2, 4)$.

9. $y = \frac{\sqrt{2x-1}}{x}$, $x > \frac{1}{2}$, at the point where $x = 2$.

Let $u = \sqrt{2x-1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2 = (2x-1)^{-\frac{1}{2}}$

Let $v = x \Rightarrow \frac{dv}{dx} = 1$

using the quotient rule: $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{x(2x-1)^{-\frac{1}{2}} - (2x-1)^{\frac{1}{2}} \times 1}{x^2}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}} - \sqrt{3}}{4}$$

$$= \frac{2-3}{4\sqrt{3}}$$

$$= \frac{-1}{4\sqrt{3}} = -\frac{1}{12}\sqrt{3}$$

10. (i) $w = \frac{1+x}{1-x}$

Let $u = 1+x \Rightarrow \frac{du}{dx} = 1$

Let $v = 1-x \Rightarrow \frac{dv}{dx} = -1$

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using the quotient rule: $\frac{dw}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$$= \frac{(1-x) - (1+x)(-1)}{(1-x)^2}$$
$$= \frac{2}{(1-x)^2}$$

(ii) $y = \left(\frac{1+x}{1-x}\right)^3 = w^3$

$$\frac{dy}{dw} = 3w^2$$

using the chain rule: $\frac{dy}{dx} = \frac{dy}{dw} \times \frac{dw}{dx}$

$$= 3w^2 \times \frac{2}{(1-x)^2}$$
$$= 3 \left(\frac{1+x}{1-x}\right)^2 \times \frac{2}{(1-x)^2}$$
$$= \frac{6(1+x)^2}{(1-x)^4}$$