Section 3: The product and quotient rules

Solutions to Exercise level 2

1. (i) $y = x(2x-1)^2$

Let $u = x \Rightarrow \frac{du}{dx} = 1$

Let $v = (2x-1)^2 \Rightarrow \frac{dv}{dx} = 2(2x-1)\times 2 = 4(2x-1)$

Use the chain rule here

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x\times 4(2x-1) + 1\times(2x-1)^2$

$= 4x(2x-1) + (2x-1)^2$

$= (2x-1)(4x + (2x-1))$

$= (2x-1)(6x-1)$

$(2x-1)$ is a common factor

(ii) $y = x(2x-1)^2 = x(4x^2 - 4x + 1) = 4x^3 - 4x^2 + x$

$\frac{dy}{dx} = 12x^2 - 8x + 1$

(iii) $(2x-1)(6x-1) = 12x^2 - 8x + 1$

so the two answers are algebraically equivalent.

2. (i) $y = \frac{5x^2}{\sqrt{x+3}}$

Let $u = 5x^2 \Rightarrow \frac{du}{dx} = 10x$

Let $v = (x+3)^{\frac{1}{2}} \Rightarrow \frac{dv}{dx} = \frac{1}{2}(x+3)^{-\frac{1}{2}}$

Using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{dy}{dx} = \frac{(x+3)^{\frac{1}{2}}\times 10x - 5x^2\times\frac{1}{2}(x+3)^{-\frac{1}{2}}}{x+3}$

$= \frac{(x+3)\times 20x - 5x^2}{2(x+3)^{\frac{3}{2}}}$

$= \frac{15x^2 + 60x}{2(x+3)^{\frac{3}{2}}}$

$= \frac{15x(x+4)}{2(x+3)^{\frac{3}{2}}}$

(ii) $y = 5x^2(x+3)^{-\frac{1}{2}}$

Let $u = 5x^2 \Rightarrow \frac{du}{dx} = 10x$

Let $v = (x+3)^{-\frac{1}{2}} \Rightarrow \frac{dv}{dx} = -\frac{1}{2}(x+3)^{-\frac{3}{2}}$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\frac{dy}{dx} = 5x^2 \times -\tfrac{1}{2}(x+3)^{-\frac{3}{2}} + (x+3)^{-\frac{1}{2}} \times 10x$$

$$= \frac{5x^2 \times -\frac{1}{2} + (x+3) \times 10x}{(x+3)^{\frac{3}{2}}}$$

$$= \frac{-5x^2 + (x+3) \times 20x}{2(x+3)^{\frac{3}{2}}}$$

$$= \frac{15x^2 + 60x}{2(x+3)^{\frac{3}{2}}}$$

$$= \frac{15x(x+4)}{2(x+3)^{\frac{3}{2}}}$$

(iii) Shown above

(iv) For $x = -3$ the denominator is zero so the function is undefined. For $x < -3$ the square root of a negative number is required so the function is undefined.

3. $y = x\sqrt{2x+5}$ for $x \geq -2.5$

Let $u = x \Rightarrow \frac{du}{dx} = 1$

Let $v = (2x+5)^{\frac{1}{2}} \Rightarrow \frac{dv}{dx} = \frac{1}{2}(2x+5)^{-\frac{1}{2}} \times 2$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\frac{dy}{dx} = x \times \frac{1}{2}(2x+5)^{-\frac{1}{2}} \times 2 + 1 \times (2x+5)^{\frac{1}{2}}$$

$$= \frac{x + 2x + 5}{(2x+5)^{\frac{1}{2}}}$$

$$= \frac{3x+5}{\sqrt{2x+5}}$$

So $a = 3$ and $b = 5$

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4. $y = \dfrac{x}{\sqrt{3x-2}}$ for $x > \frac{2}{3}$

Let $u = x \Rightarrow \dfrac{du}{dx} = 1$

Let $v = (3x-2)^{\frac{1}{2}} \Rightarrow \dfrac{dv}{dx} = \dfrac{1}{2}(3x-2)^{-\frac{1}{2}} \times 3$

Using the quotient rule: $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{(3x-2)^{\frac{1}{2}} \times 1 - x \times \frac{1}{2}(3x-2)^{-\frac{1}{2}} \times 3}{3x-2}$$

$$= \frac{(3x-2) - x \times \frac{3}{2}}{(3x-2)^{\frac{3}{2}}}$$

$$= \frac{\frac{3}{2}x - 2}{(3x-2)^{\frac{3}{2}}}$$

So $a = \frac{3}{2}$ and $b = -2$

5. $y = x\sqrt{1-x} = x(1-x)^{\frac{1}{2}}$

Use the chain rule here

Let $u = x \Rightarrow \dfrac{du}{dx} = 1$

Let $v = (1-x)^{\frac{1}{2}} \Rightarrow \dfrac{dv}{dx} = \frac{1}{2}(1-x)^{-\frac{1}{2}} \times -1 = -\dfrac{1}{2\sqrt{1-x}}$

Using the product rule: $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$$= x \times -\frac{1}{2\sqrt{1-x}} + \sqrt{1-x} \times 1$$

$$= -\frac{x}{2\sqrt{1-x}} + \sqrt{1-x}$$

At turning point, $\dfrac{x}{2\sqrt{1-x}} = \sqrt{1-x}$

$$x = 2(1-x)$$

$$x = 2 - 2x$$

$$3x = 2$$

$$x = \frac{2}{3}$$

When $x = \frac{2}{3}$, $y = \frac{2}{3}(1-\frac{2}{3})^{\frac{1}{2}} = \dfrac{2}{3\sqrt{3}}$

Turning point is $\left(\dfrac{2}{3}, \dfrac{2}{3\sqrt{3}}\right)$.

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6. $y = \dfrac{x}{2x-1}$

Let $u = x \quad \Rightarrow \dfrac{du}{dx} = 1$

Let $v = 2x-1 \quad \Rightarrow \dfrac{dv}{dx} = 2$

Using the quotient rule, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$$= \frac{1(2x-1) - 2x}{(2x-1)^2}$$

$$= -\frac{1}{(2x-1)^2}$$

Since $(2x-1)^2$ is always positive, $-\dfrac{1}{(2x-1)^2}$ is always negative for $x \neq \frac{1}{2}$, and so the gradient of the curve is always negative for $x \neq \frac{1}{2}$.

7. $y = x^2\sqrt{2x-1}$, $x > \frac{1}{2}$, at the point where $x = 1$

Let $u = x^2 \Rightarrow \dfrac{du}{dx} = 2x$

Let $v = (2x-1)^{\frac{1}{2}} \Rightarrow \dfrac{dv}{dx} = \dfrac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2 = (2x-1)^{-\frac{1}{2}}$

Using the product rule: $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$$\frac{dy}{dx} = x^2(2x-1)^{-\frac{1}{2}} + 2x(2x-1)^{\frac{1}{2}}$$

$$= 1 \times 1 + 2 \times 1$$

$$= 3$$

Gradient $= 3$, $x = 1$, $y = 1$

Equation of tangent: $y - 1 = 3(x-1)$

$$y = 3x - 2$$

8. $y = \dfrac{x^2}{x-1}$

Let $u = x^2 \quad \Rightarrow \dfrac{du}{dx} = 2x$

Let $v = x-1 \quad \Rightarrow \dfrac{dv}{dx} = 1$

Using the quotient rule, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$= \frac{2x(x-1) - x^2}{(x-1)^2}$$

$$= \frac{x(x-2)}{(x-1)^2}$$

At turning points, $\frac{x(x-2)}{(x-1)^2} = 0$

$x = 0$ or $x = 2$

When $x = 0$, $y = 0$

When $x = 2$, $y = \frac{2^2}{2-1} = 4$

The turning points are (0, 0) and (2, 4).

9. $y = \frac{\sqrt{2x-1}}{x}$, $x > \frac{1}{2}$, at the point where $x = 2$.

Let $u = \sqrt{2x-1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2 = (2x-1)^{-\frac{1}{2}}$

Let $v = x \Rightarrow \frac{dv}{dx} = 1$

Using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{x(2x-1)^{-\frac{1}{2}} - (2x-1)^{\frac{1}{2}} \times 1}{x^2}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}} - \sqrt{3}}{4}$$

$$= \frac{2-3}{4\sqrt{3}}$$

$$= \frac{-1}{4\sqrt{3}} = -\frac{1}{12}\sqrt{3}$$

10. (i) $w = \frac{1+x}{1-x}$

Let $u = 1 + x \Rightarrow \frac{du}{dx} = 1$

Let $v = 1 - x \Rightarrow \frac{dv}{dx} = -1$

Using the quotient rule: $\frac{dw}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$= \frac{(1-x)-(1+x)(-1)}{(1-x)^2}$$

$$= \frac{2}{(1-x)^2}$$

(ii) $y = \left(\frac{1+x}{1-x}\right)^3 = w^3$

$\frac{dy}{dw} = 3w^3$

Using the chain rule: $\frac{dy}{dx} = \frac{dy}{dw} \times \frac{dw}{dx}$

$$= 3w^2 \times \frac{2}{(1-x)^2}$$

$$= 3\left(\frac{1+x}{1-x}\right)^2 \times \frac{2}{(1-x)^2}$$

$$= \frac{6(1+x)^2}{(1-x)^4}$$