Section 3: The product and quotient rules

Solutions to Exercise level 1

1. (i) $y = x(x-5)^3$

Let $u = x \Rightarrow \frac{du}{dx} = 1$

Let $v = (x-5)^3 \Rightarrow \frac{dv}{dx} = 3(x-5)^2$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\frac{dy}{dx} = x \times 3(x-5)^2 + (x-5)^3 \times 1$$
$$= 3x(x-5)^2 + (x-5)^3$$
$$= (x-5)^2(3x + x - 5)$$
$$= (x-5)^2(4x-5)$$

(ii) $y = x^2(4+3x)^{\frac{1}{3}}$

Let $u = x^2 \Rightarrow \frac{du}{dx} = 2x$

Let $v = (4+3x)^{\frac{1}{3}} \Rightarrow \frac{dv}{dx} = \frac{1}{3}(4+3x)^{-\frac{2}{3}} \times 3$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\frac{dy}{dx} = 2x(4+3x)^{\frac{1}{3}} + x^2 \times \frac{1}{3} \times (4+3x)^{-\frac{2}{3}} \times 3$$
$$= 2x(4+3x)^{\frac{1}{3}} + x^2(4+3x)^{-\frac{2}{3}}$$
$$= x(4+3x)^{-\frac{2}{3}}(2(4+3x) + x)$$
$$= x(4+3x)^{-\frac{2}{3}}(8+7x)$$

2. (i) $y = x^{\frac{1}{2}}(5-3x)^4$

Let $u = x^{\frac{1}{2}} \Rightarrow \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

Let $v = (5-3x)^4 \Rightarrow \frac{dv}{dx} = 4(5-3x)^3 \times -3$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\frac{dy}{dx} = x^{\frac{1}{2}} \times 4(5-3x)^3 \times -3 + (5-3x)^4 \times \frac{1}{2}x^{-\frac{1}{2}}$$
$$= -12x^{\frac{1}{2}}(5-3x)^3 + \frac{1}{2}x^{-\frac{1}{2}}(5-3x)^4$$
$$= \frac{1}{2}x^{-\frac{1}{2}}(5-3x)^3(-24x+5-3x)$$
$$= \frac{1}{2}x^{-\frac{1}{2}}(5-3x)^3(5-27x)$$

(ii) $y = x^3(5-x^2)^{\frac{1}{2}}$

Let $u = x^3 \Rightarrow \frac{du}{dx} = 3x^2$

Let $v = (5-x^2)^{\frac{1}{2}} \Rightarrow \frac{dv}{dx} = \frac{1}{2}(5-x^2)^{-\frac{1}{2}} \times -2x$

using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\frac{dy}{dx} = x^3 \times \frac{1}{2}(5-x^2)^{-\frac{1}{2}} \times -2x + (5-x^2)^{\frac{1}{2}} \times 3x^2$$
$$= -x^4(5-x^2)^{-\frac{1}{2}} + 3x^2(5-x^2)^{\frac{1}{2}}$$
$$= x^2(5-x^2)^{-\frac{1}{2}}\left(-x^2 + 3(5-x^2)\right)$$
$$= \frac{x^2}{\sqrt{(5-x^2)}}(15-4x^2)$$

3. (i) $y = \frac{3x}{2x^2-5}$

Let $u = 3x \Rightarrow \frac{du}{dx} = 3$

Let $v = 2x^2 - 5 \Rightarrow \frac{dv}{dx} = 4x$

using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{(2x^2-5)\times 3 - 3x \times 4x}{(2x^2-5)^2}$$
$$= \frac{-6x^2-15}{(2x^2-5)^2}$$
$$= \frac{-3(2x^2+5)}{(2x^2-5)^2}$$

(ii) $y = \frac{3x}{4+\sqrt{x}}$

Let $u = 3x \Rightarrow \frac{du}{dx} = 3$

Let $v = 4 + \sqrt{x} \Rightarrow \frac{dv}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

Using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{(4+\sqrt{x})\times 3 - 3x \times \frac{1}{2}x^{-\frac{1}{2}}}{(4+\sqrt{x})^2}$$

$$= \frac{12 + 3\sqrt{x} - \frac{3}{2}\sqrt{x}}{(4+\sqrt{x})^2}$$

$$= \frac{12 + \frac{3}{2}\sqrt{x}}{(4+\sqrt{x})^2}$$

4. (i) $y = \frac{\sqrt{x}}{(3x+4)^2}$

Let $u = x^{\frac{1}{2}} \Rightarrow \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

Let $v = (3x+4)^2 \Rightarrow \frac{dv}{dx} = 2(3x+4)\times 3 = 6(3x+4)$

Using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{(3x+4)^2 \times \frac{1}{2}x^{-\frac{1}{2}} - x^{\frac{1}{2}} \times 6(3x+4)}{(3x+4)^4}$$

$$= \frac{(3x+4)\frac{1}{2}x^{-\frac{1}{2}}(3x+4-12x)}{(3x+4)^4}$$

$$= \frac{4-9x}{2\sqrt{x}(3x+4)^3}$$

(ii) $y = \frac{x}{\sqrt{3x-2}}$

Let $u = x \Rightarrow \frac{du}{dx} = 1$

Let $v = \sqrt{3x-2} \Rightarrow \frac{dv}{dx} = \frac{1}{2}(3x-2)^{-\frac{1}{2}} \times 3$

Using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{(3x-2)^{\frac{1}{2}} \times 1 - x \times \frac{1}{2}(3x-2)^{-\frac{1}{2}} \times 3}{3x-2}$$
$$= \frac{(3x-2) - \frac{3}{2}x}{(3x-2)^{\frac{1}{2}}(3x-2)}$$
$$= \frac{\frac{3}{2}x - 2}{(3x-2)^{\frac{3}{2}}}$$

5. (i) $y = \frac{7x-3}{\sqrt{2x+1}}$

Let $u = 7x-3 \Rightarrow \frac{du}{dx} = 7$

Let $v = \sqrt{2x+1} \Rightarrow \frac{dv}{dx} = \frac{1}{2}(2x+1)^{-\frac{1}{2}} \times 2 = (2x+1)^{-\frac{1}{2}}$

Using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{(2x+1)^{\frac{1}{2}} \times 7 - (7x-3) \times (2x+1)^{-\frac{1}{2}}}{2x+1}$$
$$= \frac{7(2x+1) - (7x-3)}{(2x+1)^{\frac{3}{2}}}$$
$$= \frac{7x+10}{(2x+1)^{\frac{3}{2}}}$$

(ii) $y = (7x-3)\sqrt{2x+1}$

Let $u = 7x-3 \Rightarrow \frac{du}{dx} = 7$

Let $v = (2x+1)^{\frac{1}{2}} \Rightarrow \frac{dv}{dx} = \frac{1}{2}(2x+1)^{-\frac{1}{2}} \times 2$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\frac{dy}{dx} = (7x-3) \times \frac{1}{2}(2x+1)^{-\frac{1}{2}} \times 2 + (2x+1)^{\frac{1}{2}} \times 7$$
$$= (2x+1)^{-\frac{1}{2}}(7x-3+7(2x+1))$$
$$= \frac{21x+4}{\sqrt{2x+1}}$$

6. $y = x(2x+1)^4$

Let $u = x \Rightarrow \frac{du}{dx} = 1$

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Let $v = (2x+1)^4 \quad \Rightarrow \frac{dv}{dx} = 4(2x+1)^3 \times 2 = 8(2x+1)^3$

$(2x + 1)^3$ is a common factor

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x \times 8(2x+1)^3 + 1 \times (2x+1)^4$

$= 8x(2x+1)^3 + (2x+1)^4$

$= (2x+1)^3(8x + (2x+1))$

$= (2x+1)^3(10x+1)$

7. $y = x\sqrt{1+2x} = x(1+2x)^{\frac{1}{2}}$

Use the chain rule here

Let $u = x \quad \Rightarrow \frac{du}{dx} = 1$

Let $v = (1+2x)^{\frac{1}{2}} \quad \Rightarrow \frac{dv}{dx} = \frac{1}{2}(1+2x)^{-\frac{1}{2}} \times 2 = \frac{1}{\sqrt{1+2x}}$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x \times \frac{1}{\sqrt{1+2x}} + 1 \times \sqrt{1+2x}$

$= \frac{x}{\sqrt{1+2x}} + \sqrt{1+2x}$

Use a common denominator to write this as a single fraction

$= \frac{x + (1+2x)}{\sqrt{1+2x}}$

$= \frac{1+3x}{\sqrt{1+2x}}$

8. $y = \frac{x^2}{\sqrt{1+x}}$

Use the chain rule here

Let $u = x^2 \quad \Rightarrow \frac{du}{dx} = 2x$

Let $v = (1+x)^{\frac{1}{2}} \quad \Rightarrow \frac{dv}{dx} = \frac{1}{2}(1+x)^{-\frac{1}{2}} \times 1 = \frac{1}{2}(1+x)^{-\frac{1}{2}}$

Using the quotient rule, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{2x(1+x)^{\frac{1}{2}} - x^2 \times \frac{1}{2}(1+x)^{-\frac{1}{2}}}{1+x}$

$= \frac{4x(1+x) - x^2}{2(1+x)^{\frac{3}{2}}}$

Multiply top and bottom by $2(1+x)^{\frac{1}{2}}$

$= \frac{4x + 3x^2}{2(1+x)^{\frac{3}{2}}} = \frac{x(4+3x)}{2(1+x)^{\frac{3}{2}}}$

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9. $y = x^3(1+x)^{\frac{1}{3}}$

Let $u = x^3 \quad \Rightarrow \frac{du}{dx} = 3x^2$

Let $v = (1+x)^{\frac{1}{3}} \quad \Rightarrow \frac{dv}{dx} = \frac{1}{3}(1+x)^{-\frac{2}{3}} \times 1 = \frac{1}{3}(1+x)^{-\frac{2}{3}}$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^3 \times \frac{1}{3}(1+x)^{-\frac{2}{3}} + 3x^2 \times (1+x)^{\frac{1}{3}}$

$= \frac{1}{3}x^2(1+x)^{-\frac{2}{3}}(x + 9(1+x))$

$= \frac{1}{3}x^2(1+x)^{-\frac{2}{3}}(10x + 9)$

$\frac{1}{3}x^2(1+x)^{-\frac{2}{3}}$ is a common factor

10. $y = \frac{1+\sqrt{x}}{1-\sqrt{x}}$

Let $u = 1 + x^{\frac{1}{2}} \quad \Rightarrow \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

Let $v = 1 - x^{\frac{1}{2}} \quad \Rightarrow \frac{dv}{dx} = -\frac{1}{2}x^{-\frac{1}{2}}$

Using the quotient rule, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{\frac{1}{2}x^{-\frac{1}{2}}(1-x^{\frac{1}{2}}) - (-\frac{1}{2}x^{-\frac{1}{2}})(1+x^{\frac{1}{2}})}{(1-x^{\frac{1}{2}})^2}$

$= \frac{\frac{1}{2}x^{-\frac{1}{2}}(1-x^{\frac{1}{2}}+1+x^{\frac{1}{2}})}{(1-x^{\frac{1}{2}})^2}$

$= \frac{x^{-\frac{1}{2}}}{(1-x^{\frac{1}{2}})^2}$

$= \frac{1}{\sqrt{x}(1-\sqrt{x})^2}$