Section 2: The chain rule

Exercise level 3 solutions

1. $y=(ax+b)^3$ passes through (0, 8) so $8=b^3$

$b=2$

$\frac{dy}{dx}=3(ax+b)^2\times a=3a(ax+2)^2$

When $x=0$, gradient $=60$ so $3a\times 4=60$

$a=5$

2. (i) $y=\left(\frac{x+1}{x}\right)^4=\left(1+\frac{1}{x}\right)^4$

$\frac{dy}{dx}=4\left(1+\frac{1}{x}\right)^3\times -\frac{1}{x^2}=-\frac{4}{x^2}\left(1+\frac{1}{x}\right)^3$

At stationary point, $-\frac{4}{x^2}\left(1+\frac{1}{x}\right)^3=0\Rightarrow x=-1$

When $x=-1, y=0$ so stationary point is at (-1, 0).

(ii) $y=x\sqrt{x+1}=(x^3+x^2)^{\frac{1}{2}}$

$\frac{dy}{dx}=\frac{1}{2}(x^3+x^2)^{-\frac{1}{2}}(3x^2+2x)=\frac{x(3x+2)}{2x\sqrt{x+1}}=\frac{3x+2}{2(x+1)}$

At stationary point, $x=-\frac{2}{3}$

When $x=-\frac{2}{3}$, $y=-\frac{2}{3}\sqrt{\frac{1}{3}}=-\frac{2}{3\sqrt{3}}$

3. (i) Stretch parallel to the y-axis, scale factor 2, horizontal translation 1 unit to the right.

(ii) P = (2, 16)

(iii) $y=x^4\Rightarrow\frac{dy}{dx}=4x^3$

At $x=2$, gradient $=4\times 2^3=32$

Gradient of transformed curve $=64$

(iv) $y=2(x-1)^4\Rightarrow\frac{dy}{dx}=2\times 4(x-1)^3=8(x-1)^3$

When $x=3$, gradient $=8\times 2^3=64$