Section 2: The chain rule

Solutions to Exercise level 2

1. (i) $y = (2x-1)^3$

$$\frac{dy}{dx} = 3(2x-1)^2 \times 2 = 6(2x-1)^2$$

(ii) $y = (2x-1)^3$

$$= (2x)^3 + 3(2x)^2(-1) + 3(2x)(-1)^2 + (-1)^3$$

$$= 8x^3 - 12x^2 + 6x - 1$$

$$\frac{dy}{dx} = 24x^2 - 24x + 6$$

(iii) $6(2x-1)^2 = 6(4x^2 - 4x + 1) = 24x^2 - 24x + 6$
so the two expressions are equivalent.

2. (i) $y = (2x+1)^4$

$$\frac{dy}{dx} = 4(2x+1)^3 \times 2 = 8(2x+1)^3$$

When $x = 0$, $\frac{dy}{dx} = 8 \times 1^3 = 8$

(ii) When the gradient is zero, $8(2x+1)^3 = 0$

$$x = -\tfrac{1}{2}$$

When $x = -\frac{1}{2}$, $y = (2 \times -\frac{1}{2} + 1)^4 = 0$

The gradient is zero at the point $(-\frac{1}{2}, 0)$.

3. $y = 6(2x-1)^{-1}$

$$\frac{dy}{dx} = -6(2x-1)^{-2} \times 2$$

$$= \frac{-12}{(2x-1)^2}$$

When $x = 2$, $\frac{dy}{dx} = -\frac{4}{3}$

Equation of tangent: $y - y_1 = m(x - x_1)$

$$y - 2 = -\tfrac{4}{3}(x-2)$$

$$3y + 4x = 14$$

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4. $y = (5x^2 + 16)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(5x^2 + 16)^{-\frac{1}{2}} \times 10x$

$= \frac{5x}{\sqrt{(5x^2 + 16)}}$

When $x = 2$, $\frac{dy}{dx} = \frac{5}{3}$

Perpendicular gradient $= -\frac{3}{5}$

Equation of normal: $y - y_1 = m(x - x_1)$

$y - 6 = -\frac{3}{5}(x - 2)$

$5y + 3x = 36$

5. (i) $x^2 + y^2 = 25$

$y^2 = 25 - x^2$

$y = (25 - x^2)^{\frac{1}{2}}$

(ii) $\frac{dy}{dx} = \frac{1}{2}(25 - x^2)^{-\frac{1}{2}} \times -2x$

$= -\frac{x}{\sqrt{25 - x^2}}$

(iii) When $x = 4$, $\frac{dy}{dx} = -\frac{4}{\sqrt{25 - 4^2}} = -\frac{4}{\sqrt{9}} = -\frac{4}{3}$

6. $V = x^3$

$\frac{dV}{dx} = 3x^2$

When $x = 20$, $\frac{dV}{dx} = 3 \times 20^2 = 1200$

$\frac{dx}{dt} = -0.2$

Using the chain rule, $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 1200 \times -0.2 = -240$

The volume is decreasing at a rate of 240 $cm^3\ h^{-1}$.

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7. $y = 10(x^2 + 9)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = -\tfrac{1}{2} \times 10 \times (x^2 + 9)^{-\frac{3}{2}} \times 2x$$

$$= -\frac{10x}{(x^2 + 9)^{\frac{3}{2}}}$$

When $x = 4$, $\frac{dy}{dx} = -\frac{40}{125} = -\frac{8}{25}$

Equation of tangent: $y - y_1 = m(x - x_1)$

$$y - 2 = -\tfrac{8}{25}(x - 4)$$

$$25y + 8x = 82$$

The tangent cuts the x-axis at (10.25, 0) and the y-axis at (0,3.28)

8. $y = 12(6 - x)^{-1}$

$$\frac{dy}{dx} = -12 \times (6 - x)^{-2} \times -1$$

$$= \frac{12}{(6 - x)^2}$$

At P, gradient = 3

$$3 = \frac{12}{(6 - x)^2}$$

$$3(6 - x)^2 = 12$$

$$36 - 12x + x^2 = 4$$

$$x^2 - 12x + 32 = 0$$

$$(x - 8)(x - 4) = 0$$

$$x = 8, x = 4$$

Coordinates of P are (8,-6) or (4, 6).

9. $\frac{dV}{dt} = 30$ cm³ per second

$V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dr} = 3 \times \tfrac{4}{3}\pi r^2 = 4\pi r^2$$

Using the chain rule: $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$

$$= \frac{1}{4\pi r^2} \times 30$$

$r = 10$ so $\frac{dr}{dt} = \frac{1}{400\pi} \times 30$

$= 0.0239$ cm / second

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10. $y = (7x - 3)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \tfrac{1}{2}(7x-3)^{-\frac{1}{2}} \times 7$$

$$= \frac{3.5}{\sqrt{(7x-3)}}$$

When $x = 4$, $\frac{dy}{dx} = \frac{7}{10}$

Perpendicular gradient $= -\frac{10}{7}$

Equation of normal: $y - y_1 = m(x - x_1)$ at the point (4, 5)

$$y - 5 = -\tfrac{10}{7}(x - 4)$$

$$y = 0, x = 7.5$$

P is (7.5, 0)