

## Section 2: The chain rule

## Solutions to Exercise level 1

1. (i)  $y = (5x - 2)^5$

Let  $u = 5x - 2 \Rightarrow \frac{du}{dx} = 5$

$y = u^5 \Rightarrow \frac{dy}{du} = 5u^4$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= 5u^4 \times 5$$

$$= 25u^4$$

$$= 25(5x - 2)^4$$

(ii)  $y = (2 + 5x)^{-1}$

$u = 2 + 5x \Rightarrow \frac{du}{dx} = 5$

$y = u^{-1} \Rightarrow \frac{dy}{du} = -u^{-2}$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= -u^{-2} \times 5$$

$$= -5u^{-2}$$

$$= -5(2 + 5x)^{-2}$$

(iii)  $y = (1 - 3x)^7$

Let  $u = 1 - 3x \Rightarrow \frac{du}{dx} = -3$

$y = u^7 \Rightarrow \frac{dy}{du} = 7u^6$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= 7u^6 \times -3$$

$$= -21u^6$$

$$= -21(1 - 3x)^6$$

(iv)  $y = (1 - 2x)^{-3}$

Let  $u = 1 - 2x \Rightarrow \frac{du}{dx} = -2$

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$$y = u^{-3} \Rightarrow \frac{dy}{du} = -3u^{-4} = -\frac{3}{u^4}$$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= -\frac{3}{u^4} \times -2$$
$$= \frac{6}{(1-2x)^4}$$

2. (i)  $(x^2 + 1)^4$

Let  $u = x^2 + 1 \Rightarrow \frac{du}{dx} = 2x$

$$y = u^4 \Rightarrow \frac{dy}{du} = 4u^3$$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= 4u^3 \times 2x$$
$$= 8u^3 \times x$$
$$= 8x(x^2 + 1)^3$$

(ii)  $(3x^2 + 5)^{-3}$

Let  $u = 3x^2 + 5 \Rightarrow \frac{du}{dx} = 6x$

$$y = u^{-3} \Rightarrow \frac{dy}{du} = -3u^{-4}$$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= -3u^{-4} \times 6x$$
$$= -18u^{-4} \times x$$
$$= -18x(3x^2 + 5)^{-4}$$

(iii)  $(5 - x^3)^4$

Let  $u = 5 - x^3 \Rightarrow \frac{du}{dx} = -3x^2$

$$y = u^4 \Rightarrow \frac{dy}{du} = 4u^3$$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= 4u^3 \times -3x^2$$
$$= -12u^3 \times x^2$$
$$= -12x^2(5 - x^3)^3$$

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(iv)  $(7 - 4x^2)^{-1}$

$$\text{Let } u = 7 - 4x^2 \Rightarrow \frac{du}{dx} = -8x$$

$$y = u^{-1} \Rightarrow \frac{dy}{du} = -u^{-2}$$

$$\begin{aligned} \text{using the chain rule: } \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= -u^{-2} \times -8x \\ &= 8u^{-2} \times x \\ &= 8x(7 - 4x^2)^{-2} \end{aligned}$$

3. (i)  $y = (5x - 2)^{\frac{1}{2}}$

$$\text{Let } u = 5x - 2 \Rightarrow \frac{du}{dx} = 5$$

$$y = u^{\frac{1}{2}} \Rightarrow \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$$

$$\begin{aligned} \text{using the chain rule: } \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= \frac{1}{2}u^{-\frac{1}{2}} \times 5 \\ &= \frac{5}{2}u^{-\frac{1}{2}} \\ &= \frac{5}{2}(5x - 2)^{-\frac{1}{2}} \end{aligned}$$

(ii)  $y = (2 - 5x)^{-\frac{1}{3}}$

$$\text{Let } u = 2 - 5x \Rightarrow \frac{du}{dx} = -5$$

$$y = u^{-\frac{1}{3}} \Rightarrow \frac{dy}{du} = -\frac{1}{3}u^{-\frac{4}{3}}$$

$$\begin{aligned} \text{using the chain rule: } \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= -\frac{1}{3}u^{-\frac{4}{3}} \times -5 \\ &= \frac{5}{3}u^{-\frac{4}{3}} \\ &= \frac{5}{3}(2 - 5x)^{-\frac{4}{3}} \end{aligned}$$

(iii)  $y = (2 + 3x)^{-\frac{2}{3}}$

$$\text{Let } u = 2 + 3x \Rightarrow \frac{du}{dx} = 3$$

$$y = u^{-\frac{2}{3}} \Rightarrow \frac{dy}{du} = -\frac{2}{3}u^{-\frac{5}{3}}$$

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using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= -\frac{2}{3}u^{-\frac{5}{3}} \times 3$$
$$= -2u^{-\frac{5}{3}}$$
$$= -2(2+3x)^{-\frac{5}{3}}$$

(iv)  $y = (1-2x)^{\frac{3}{2}}$

Let  $u = 1-2x \Rightarrow \frac{du}{dx} = -2$

$y = u^{\frac{3}{2}} \Rightarrow \frac{dy}{du} = \frac{3}{2}u^{\frac{1}{2}}$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= \frac{3}{2}u^{\frac{1}{2}} \times -2$$
$$= -3u^{\frac{1}{2}}$$
$$= -3(1-2x)^{\frac{1}{2}}$$

4. (i)  $(3x^2+1)^{\frac{4}{3}}$

Let  $u = 3x^2+1 \Rightarrow \frac{du}{dx} = 6x$

$y = u^{\frac{4}{3}} \Rightarrow \frac{dy}{du} = \frac{4}{3}u^{\frac{1}{3}}$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= \frac{4}{3}u^{\frac{1}{3}} \times 6x$$
$$= 8u^{\frac{1}{3}} \times x$$
$$= 8x(3x^2+1)^{\frac{1}{3}}$$

(ii)  $(3-2x^2)^{\frac{3}{5}}$

Let  $u = 3-2x^2 \Rightarrow \frac{du}{dx} = -4x$

$y = u^{\frac{3}{5}} \Rightarrow \frac{dy}{du} = \frac{3}{5}u^{-\frac{2}{5}}$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= \frac{3}{5}u^{-\frac{2}{5}} \times -4x$$
$$= -\frac{12}{5}u^{-\frac{2}{5}} \times x$$
$$= -\frac{12}{5}x(3-2x^2)^{-\frac{2}{5}}$$

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$$(iii) (5+2x^3)^{-\frac{3}{2}}$$

$$\text{Let } u = 5 + 2x^3 \Rightarrow \frac{du}{dx} = 6x^2$$

$$y = u^{-\frac{3}{2}} \Rightarrow \frac{dy}{du} = -\frac{3}{2}u^{-\frac{5}{2}}$$

$$\begin{aligned} \text{using the chain rule: } \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= -\frac{3}{2}u^{-\frac{5}{2}} \times 6x^2 \\ &= -9u^{-\frac{5}{2}} \times x^2 \\ &= -9x^2(5+2x^3)^{-\frac{5}{2}} \end{aligned}$$

$$(iv) (5-2x^2)^{-\frac{2}{5}}$$

$$\text{Let } u = 5 - 2x^2 \Rightarrow \frac{du}{dx} = -4x$$

$$y = u^{-\frac{2}{5}} \Rightarrow \frac{dy}{du} = -\frac{2}{5}u^{-\frac{7}{5}}$$

$$\begin{aligned} \text{using the chain rule: } \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= -\frac{2}{5}u^{-\frac{7}{5}} \times -4x \\ &= \frac{8}{5}u^{-\frac{7}{5}} \times x \\ &= \frac{8}{5}x(5-2x^2)^{-\frac{7}{5}} \end{aligned}$$

$$5. (i) \sqrt{6x-5}$$

$$y = (6x-5)^{\frac{1}{2}}$$

$$\text{Let } u = 6x - 5 \Rightarrow \frac{du}{dx} = 6$$

$$y = u^{\frac{1}{2}} \Rightarrow \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$$

$$\begin{aligned} \text{using the chain rule: } \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= \frac{1}{2}u^{-\frac{1}{2}} \times 6 \\ &= 3u^{-\frac{1}{2}} \\ &= 3(6x-5)^{-\frac{1}{2}} \\ &= \frac{3}{\sqrt{6x-5}} \end{aligned}$$

$$(ii) \sqrt[3]{(x^2-2)}$$

$$y = (x^2-2)^{\frac{1}{3}}$$

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$$\text{Let } u = x^2 - 2 \Rightarrow \frac{du}{dx} = 2x$$

$$y = u^{\frac{1}{3}} \Rightarrow \frac{dy}{du} = \frac{1}{3}u^{-\frac{2}{3}}$$

$$\begin{aligned}\text{using the chain rule: } \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= \frac{1}{3}u^{-\frac{2}{3}} \times 2x \\ &= \frac{2}{3}u^{-\frac{2}{3}} \times x \\ &= \frac{2}{3}x(x^2 - 2)^{-\frac{2}{3}} \\ &= \frac{\frac{2}{3}x}{(x^2 - 2)^{\frac{2}{3}}}\end{aligned}$$

$$6. \text{ (i) } \frac{1}{3x-2}$$

$$y = (3x-2)^{-1}$$

$$\text{Let } u = 3x-2 \Rightarrow \frac{du}{dx} = 3$$

$$y = u^{-1} \Rightarrow \frac{dy}{du} = -u^{-2}$$

$$\begin{aligned}\text{using the chain rule: } \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= -u^{-2} \times 3 \\ &= -3u^{-2} \\ &= -3(3x-2)^{-2} \\ &= -\frac{3}{(3x-2)^2}\end{aligned}$$

$$(ii) \frac{5}{x^2-4x-3}$$

$$y = 5(x^2-4x-3)^{-1}$$

$$\text{Let } u = x^2 - 4x - 3 \Rightarrow \frac{du}{dx} = 2x - 4$$

$$y = 5u^{-1} \Rightarrow \frac{dy}{du} = -5u^{-2}$$

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using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\begin{aligned} &= 5 \times -u^{-2} \times (2x-4) \\ &= -5u^{-2} \times (2x-4) \\ &= -10(x-2)(x^2-4x-3)^{-2} \\ &= -\frac{10(x-2)}{(x^2-4x-3)^2} \end{aligned}$$

7. (i)  $\frac{1}{\sqrt{x^3+3x}}$

$$y = (x^3 + 3x)^{-\frac{1}{2}}$$

$$\text{Let } u = x^3 + 3x \Rightarrow \frac{du}{dx} = 3x^2 + 3$$

$$y = u^{-\frac{1}{2}} \Rightarrow \frac{dy}{du} = -\frac{1}{2}u^{-\frac{3}{2}}$$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\begin{aligned} &= -\frac{1}{2}u^{-\frac{3}{2}} \times (3x^2 + 3) \\ &= -\frac{3}{2}u^{-\frac{3}{2}} \times (x^2 + 1) \\ &= -\frac{3}{2}(x^3 + 3x)^{-\frac{3}{2}} \times (x^2 + 1) \\ &= -\frac{\frac{3}{2}(x^2 + 1)}{(x^3 + 3x)\sqrt{x^3 + 3x}} \end{aligned}$$

(ii)  $\frac{3}{\sqrt[3]{x^2+1}}$

$$y = 3(x^2 + 1)^{-\frac{1}{3}}$$

$$\text{Let } u = x^2 + 1 \Rightarrow \frac{du}{dx} = 2x$$

$$y = 3u^{-\frac{1}{3}} \Rightarrow \frac{dy}{du} = -3 \times \frac{1}{3}u^{-\frac{4}{3}}$$

using the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\begin{aligned} &= 3 \times -\frac{1}{3}u^{-\frac{4}{3}} \times 2x \\ &= -2u^{-\frac{4}{3}} \times x \\ &= -2x(x^2 + 1)^{-\frac{4}{3}} \\ &= -\frac{2x}{(x^2 + 1)^{\frac{4}{3}}} \end{aligned}$$

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$$8. y = \frac{1}{2x-1} = (2x-1)^{-1}$$

$$\frac{dy}{dx} = -(2x-1)^{-2} \times 2 = -\frac{2}{(2x-1)^2}$$

$$\text{When } x = 1, \frac{dy}{dx} = -\frac{2}{(2-1)^2} = -2.$$

$$9. y = (3x^2 - 3x - 2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} \times (3x^2 - 3x - 2)^{-\frac{1}{2}} \times (6x - 3)$$

$$= \frac{\frac{1}{2}(6x-3)}{\sqrt{(3x^2-3x-2)}}$$

$$\text{When } x = 2, \frac{dy}{dx} = \frac{\frac{1}{2} \times 9}{\sqrt{(3 \times 2^2 - 3 \times 2 - 2)}} \\ = 2.25$$

$$10. y = (2x-1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -\frac{1}{2} \times (2x-1)^{-\frac{3}{2}} \times 2$$

$$= -\frac{1}{(2x-1)^{\frac{3}{2}}}$$

$$\text{When } x = 1, \frac{dy}{dx} = -\frac{1}{(2 \times 1 - 1)^{\frac{3}{2}}} \\ = -1$$