Section 1: Differentiating exponentials and logarithms

Solutions to Exercise level 1

1. (i) $\frac{dy}{dx} = 4e^{4x}$

(ii) $\frac{dy}{dx} = -2e^{-2x}$

(iii) $\frac{dy}{dx} = \frac{1}{2}e^{\frac{1}{2}x}$

2. Let $u = 2x \Rightarrow \frac{du}{dx} = 2$

$y = \ln 2x = \ln u \Rightarrow \frac{dy}{du} = \frac{1}{u}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{u} \times 2 = \frac{1}{2x} \times 2 = \frac{1}{x}$

so the derivative of ln 2x is the same as the derivative of ln x.

$y = \ln 2x = \ln 2 + \ln x$

$\frac{dy}{dx} = \frac{1}{x}$ since ln 2 is a constant and therefore has derivative zero.

3. (i) $\frac{dy}{dx} = \frac{1}{x}$

(ii) $\frac{dy}{dx} = \frac{1}{x}$

(iii) $\frac{dy}{dx} = \frac{1}{x}$

4. (i) $y = xe^{2x}$

Let $u = x \Rightarrow \frac{du}{dx} = 1$

Let $v = e^{2x} \Rightarrow \frac{dv}{dx} = 2e^{2x}$

Using the product rule, $\frac{dy}{dx} = x \times 2e^{2x} + 1 \times e^{2x}$

$= (2x+1)e^{2x}$

(ii) $y = \frac{e^{-x}}{2x+1}$

Let $u = e^{-x} \Rightarrow \frac{du}{dx} = -e^{-x}$

Let $v = 2x+1 \Rightarrow \frac{dv}{dx} = 2$

Using the quotient rule: $\frac{dy}{dx} = \frac{(2x+1) \times -e^{-x} - 2e^{-x}}{(2x+1)^2}$

$= \frac{-(2x+3)e^{-x}}{(2x+1)^2}$

(iii) $y = e^{2x-x^2}$

Let $u = 2x - x^2 \Rightarrow \frac{du}{dx} = 2 - 2x$

$y = e^u \Rightarrow \frac{dy}{du} = e^u$

Using the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^u(2-2x)$

$= (2-2x)e^{2x-x^2}$

5. (i) $y = x^2 \ln x$

Let $u = x^2 \Rightarrow \frac{du}{dx} = 2x$

Let $v = \ln x \Rightarrow \frac{dv}{dx} = \frac{1}{x}$

Using the product rule, $\frac{dy}{dx} = x^2 \times \frac{1}{x} + 2x \ln x$

$= x + 2x \ln x$

(ii) $y = \frac{\ln x}{1+x}$

Let $u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$

Let $v = 1 + x \Rightarrow \frac{dv}{dx} = 1$

Using the quotient rule, $\frac{dy}{dx} = \frac{(1+x) \times \frac{1}{x} - \ln x \times 1}{(1+x)^2} = \frac{1 + x - x \ln x}{x(1+x)^2}$

(iii) $y = \ln(1 + x^2)$

Let $u = 1 + x^2 \quad \Rightarrow \frac{du}{dx} = 2x$

$y = \ln u \quad \Rightarrow \frac{dy}{du} = \frac{1}{u}$

Using the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{u} \times 2x = \frac{2x}{1 + x^2}$