

Section 2: Differentiating trigonometric functions

Solutions to Exercise level 3

$$\begin{aligned}
 1. \quad (i) \quad f(x) &= \sin^2 x + \cos^2 x \\
 f'(x) &= 2 \sin x \times \cos x + 2 \cos x \times -\sin x \\
 &= 2 \sin x \cos x - 2 \sin x \cos x \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad f'(x) &= 0 \Rightarrow f(x) = k \\
 f(0) &= \sin^2 0 + \cos^2 0 = 0 + 1 = 1 \\
 \text{so } f(x) &= 1 \\
 \sin^2 x + \cos^2 x &= 1
 \end{aligned}$$

$$\begin{aligned}
 2. \quad (i) \quad y &= \arcsin x \\
 x &= \sin y \\
 \frac{dx}{dy} &= \cos y
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \frac{dy}{dx} &= \frac{1}{\cos y} \\
 \cos^2 y &= 1 - \sin^2 y \\
 \text{so } \cos y &= \sqrt{1 - \sin^2 y} \quad (\text{positive square root as } y = \arcsin x \text{ so} \\
 &\quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \text{ and } \cos y \geq 0 \text{ in this interval}) \\
 \frac{dy}{dx} &= \frac{1}{\sqrt{1 - \sin^2 y}} \\
 &= \frac{1}{\sqrt{1 - x^2}} \\
 \text{Hence } \int \frac{1}{\sqrt{1 - x^2}} dx &= \arcsin x + c
 \end{aligned}$$

$$\begin{aligned}
 3. \quad (i) \quad y &= \cos(\pi e^x) \\
 \frac{dy}{dx} &= -\pi e^x \sin(\pi e^x)
 \end{aligned}$$

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(ii) At stationary points, $\sin(\pi e^x) = 0$

$$\pi e^x = m\pi$$

where m is an integer, which must be non-negative as e^x is positive for all x

$$e^x = m$$

$$x = \ln m$$

(iii) Distance between stationary points $= \ln(m+1) - \ln m$

$$= \ln \frac{m+1}{m}$$

$$= \ln \left(1 + \frac{1}{m} \right)$$

As $m \rightarrow \infty$, $\frac{1}{m} \rightarrow 0$ so distance between stationary points $\rightarrow \ln 1 = 0$

(iv)

