Section 2: Differentiating trigonometric functions

Solutions to Exercise level 3

1. (i) $f(x) = \sin^2 x + \cos^2 x$

$f'(x) = 2\sin x \times \cos x + 2\cos x \times -\sin x$

$= 2\sin x \cos x - 2\sin x \cos x$

$= 0$

(ii) $f'(x) = 0 \Rightarrow f(x) = k$

$f(0) = \sin^2 0 + \cos^2 0 = 0 + 1 = 1$

so $f(x) = 1$

$\sin^2 x + \cos^2 x = 1$

2. (i) $y = \arcsin x$

$x = \sin y$

$\frac{dx}{dy} = \cos y$

(ii) $\frac{dy}{dx} = \frac{1}{\cos y}$

$\cos^2 y = 1 - \sin^2 y$

so $\cos y = \sqrt{1 - \sin^2 y}$ (positive square root as $y = \arcsin x$ so $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and $\cos y \ge 0$ in this interval)

$\frac{dy}{dx} = \frac{1}{\sqrt{1 - \sin^2 y}}$

$= \frac{1}{\sqrt{1 - x^2}}$

Hence $\int \frac{1}{\sqrt{1 - x^2}}\, dx = \arcsin x + c$

3. (i) $y = \cos(\pi e^x)$

$\frac{dy}{dx} = -\pi e^x \sin(\pi e^x)$

(ii) At stationary points, $\sin(\pi e^x) = 0$

$$\pi e^x = m\pi$$

where m is an integer, which must be non-negative as e^x is positive for all x

$$e^x = m$$
$$x = \ln m$$

(iii) Distance between stationary points $= \ln(m+1) - \ln m$

$$= \ln \frac{m+1}{m}$$
$$= \ln\left(1 + \frac{1}{m}\right)$$

As $m \to \infty$, $\frac{1}{m} \to 0$ so distance between stationary points $\to \ln 1 = 0$

(iv)