Section 2: Differentiating trigonometric functions

Solutions to Exercise level 2

1. $f(x) = e^{2x}\tan x$

Let $u = e^{2x} \Rightarrow \frac{du}{dx} = 2e^{2x}$

Let $v = \tan x \Rightarrow \frac{dv}{dx} = \sec^2 x$

Using the product rule: $\frac{dy}{dx} = e^{2x}\sec^2 x + 2e^{2x}\tan x$

$= e^{2x}(\sec^2 x + 2\tan x)$

2. $y = \sin x + 2\cos x$

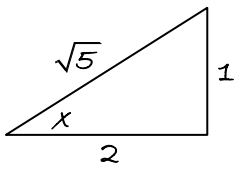
$\frac{dy}{dx} = \cos x - 2\sin x$

At stationary points, $\cos x - 2\sin x = 0$

$2\sin x = \cos x$

$\tan x = \frac{1}{2}$

For $0 < x < \frac{\pi}{2}$,



$\sin x = \frac{1}{\sqrt{5}}$ and $\cos x = \frac{2}{\sqrt{5}}$

$y = \sin x + 2\cos x$

$= \frac{1}{\sqrt{5}} + \frac{4}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$

3. $y = \sin 3x$

$\frac{dy}{dx} = 3\cos 3x$

$\frac{d^2y}{dx^2} = 3 \times -3\sin 3x = -9\sin 2x = -9y$

4. $y = \sin^3 x = (\sin x)^3$

$\frac{dy}{dx} = 3(\sin x)^2 \times \cos x = 3\sin^2 x \cos x$

Using the result from above

$$\int_0^{\frac{\pi}{2}} \sin^2 x \cos x \, dx = \tfrac{1}{3}\int_0^{\frac{\pi}{2}} 3\sin^2 x \cos x \, dx$$
$$= \tfrac{1}{3}\left[\sin^3 x\right]_0^{\frac{\pi}{2}}$$
$$= \tfrac{1}{3}(1^3 - 0^3)$$
$$= \tfrac{1}{3}$$

5. (i) $x = 5\sin\frac{1}{2}t$

When $t = 2$, $x = 5\sin 1 = 4.21$

The height of the water is 4.21 m.

The calculator must be set to radians

$$\frac{dx}{dt} = \tfrac{5}{2}\cos\tfrac{1}{2}t$$

When $t = 2$, $\frac{dx}{dt} = \tfrac{5}{2}\cos 1 = 1.35$

Rate at which the water is rising is 1.35 metres / hour.

The value of $\frac{dx}{dt}$ is greatest when $\cos\frac{1}{2}t = 1 \Rightarrow \frac{1}{2}t = 0 \Rightarrow t = 0$

The water is rising most rapidly at noon, when it is rising at a rate of 2.5 metres / hour.

6. (i) $y = 0.5 + 0.2\sin(10t)$

Greatest height $= 0.5 + 0.2 = 0.7$ metres

Least height $= 0.5 - 0.2 = 0.3$ metres

The calculator must be set to radians

(ii) $\frac{dy}{dt} = 0.2 \times 10\cos(10t) = 2\cos(10t)$

When $t = 0.5$, $\frac{dy}{dt} = 2\cos(10 \times 0.5) = 2\cos 5 = 0.567\ ms^{-1}$.