Section 2: Differentiating trigonometric functions

Solutions to Exercise level 1

1. (i) $y = \sin 4x \quad \Rightarrow \frac{dy}{dx} = 4\cos 4x$

(ii) $y = \cos 3x \quad \Rightarrow \frac{dy}{dx} = -3\sin 3x$

(iii) $y = \tan 2x \quad \Rightarrow \frac{dy}{dx} = 2\sec^2 2x$

2. (i) $y = x^2 \sin x$

Let $u = x^2 \quad \Rightarrow \frac{du}{dx} = 2x$

Let $v = \sin x \quad \Rightarrow \frac{dv}{dx} = \cos x$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^2 \cos x + 2x\sin x$

(ii) $y = \frac{\cos x}{1+x}$

Let $u = \cos x \quad \Rightarrow \frac{du}{dx} = -\sin x$

Let $v = 1 + x \quad \Rightarrow \frac{dv}{dx} = 1$

Using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(1+x)\times -\sin x - \cos x \times 1}{(1+x)^2}$

$= \frac{-(1+x)\sin x - \cos x}{(1+x)^2}$

(iii) $y = \sin(1+x^2)$

Let $u = 1 + x^2 \quad \Rightarrow \frac{du}{dx} = 2x$

$y = \sin u \quad \Rightarrow \frac{dy}{du} = \cos u$

Using the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos u \times 2x$

$= 2x\cos(1+x^2)$

(iv) $y = \sin^2 x$

Let $u = \sin x \quad \Rightarrow \frac{du}{dx} = \cos x$

$y = u^2 \quad \Rightarrow \frac{dy}{du} = 2u$

Using the chain rule: $\frac{dy}{dx} = 2u\cos x = 2\sin x\cos x$

(v) $y = \cos^3 4x$

Let $u = \cos 4x \quad \Rightarrow \frac{du}{dx} = -4\sin 4x$

$y = u^3 \quad \Rightarrow \frac{dy}{du} = 3u^2$

Using the chain rule: $\frac{dy}{dx} = 3u^2 \times -4\sin 4x = -12\cos^2 4x\sin 4x$

(vi) $y = e^{\tan x}$

Let $u = \tan x \quad \Rightarrow \frac{du}{dx} = \sec^2 x$

$y = e^u \quad \Rightarrow \frac{dy}{du} = e^u$

Using the chain rule: $\frac{dy}{dx} = e^u \times \sec^2 x = e^{\tan x}\sec^2 x$

(vii) $y = \ln(\cos x)$

Let $u = \cos x \quad \Rightarrow \frac{du}{dx} = -\sin x$

$y = \ln u \quad \Rightarrow \frac{dy}{du} = \frac{1}{u}$

Using the chain rule: $\frac{dy}{dx} = \frac{1}{u} \times -\sin x = -\frac{\sin x}{\cos x} = -\tan x$

(viii) $y = e^{2x}\sin x$

Let $u = e^{2x} \quad \Rightarrow \frac{du}{dx} = 2e^{2x}$

Let $v = \sin x \quad \Rightarrow \frac{dv}{dx} = \cos x$

Using the product rule: $\frac{dy}{dx} = 2e^{2x}\sin x + e^{2x}\cos x$

$= e^{2x}(2\sin x + \cos x)$

3. $y = \cos x° = \cos\frac{\pi x}{180}$

Angles must be in radians before you can differentiate

Using the chain rule, $\frac{dy}{dx} = -\frac{\pi}{180}\sin\frac{\pi x}{180} = -\frac{\pi}{180}\sin x°$