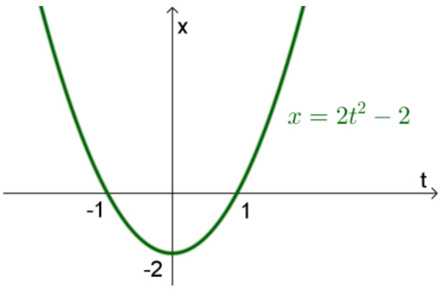
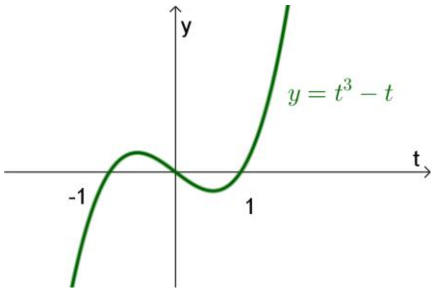
Section 1: Using parametric equations

Solutions to Exercise level 3

1. (i)





$x \geq 2, y \in \mathbb{R}$

(ii) When $y = x$, $t^3 - t = 2t^2 - 2$

$t^3 - 2t^2 - t + 2 = 0$

$1^3 - 2 \times 1^2 - 1 + 2 = 0$, so $(t-1)$ is a factor

$(t-1)(t^2 - t - 2) = 0$

$(t-1)(t+1)(t-2) = 0$

$t = 1, -1$ or 2

When $t = 1, x = 0, y = 0$

When $t = -1, x = 0, y = 0$

When $t = 2, x = 6, y = 6$

so the curve meets the line $y = x$ at $(0, 0)$ and $(6, 6)$

(iii) $x = 2t^2 - 2$

$x + 2 = 2t^2$

$t^2 = \dfrac{x+2}{2}$

$y = t^3 - t = t(t^2 - 1)$

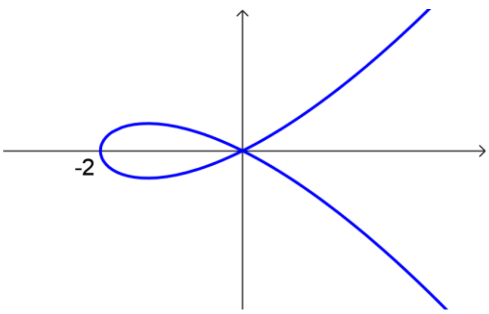
$y^2 = t^2(t^2 - 1)^2$

$= \left(\dfrac{x+2}{2}\right)\left(\dfrac{x+2}{2} - 1\right)^2$

$= \left(\dfrac{x+2}{2}\right)\left(\dfrac{x}{2}\right)^2$

$8y^2 = x^2(x+2)$

$8y^2 = x^3 + 2x^2$



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2. (i) If the curve is symmetric in the line $y = x$, then if the point (a, b) lies on the curve then so does the point (b, a).
If (a, b) lies on the curve then $a^3 + b^3 = 3ab$
so $b^3 + a^3 = 3ba$ and hence (b, a) lies on the curve.

(ii) $y = tx \Rightarrow x^3 + (tx)^3 = 3xtx$

$$\Rightarrow x^3 + t^3x^3 = 3tx^2$$

$$\Rightarrow x(1 + t^3) = 3t$$

$$\Rightarrow x = \frac{3t}{1 + t^3} \quad (t \neq -1)$$

$$y = tx = \frac{3t^2}{1 + t^3} \quad (t \neq -1)$$

(iii) Where the curve meets $y = x$, $\frac{3t}{1 + t^3} = \frac{3t^2}{1 + t^3}$

$$t^2 - t = 0$$

$$t(t - 1) = 0$$

$$t = 0 \text{ or } 1$$

When $t = 0, x = 0, y = 0$
When $t = 1, x = \frac{3}{2}, y = \frac{3}{2}$
The curve meets the line $y = x$ at (0, 0) and $\left(\frac{3}{2}, \frac{3}{2}\right)$

(iii) $x^3 + y^3 = 3xy$

Differentiating implicitly: $3x^2 + 3y^2 \frac{dy}{dx} = 3y + 3x \frac{dy}{dx}$

When $\frac{dy}{dx} = 0$, $3x^2 = 3y$

$$x^2 = y$$

$$\left(\frac{3t}{1 + t^3}\right)^2 = \frac{3t^2}{1 + t^3}$$

$$\frac{9t^2}{(1 + t^3)^2} = \frac{3t^2}{1 + t^3}$$

$$3t^2 = t^2(1 + t^3)$$

$$t^2(t^3 - 2) = 0$$

$$t = 0 \text{ or } 2^{\frac{1}{3}}$$

As $t = 0$ corresponds to the origin, the maximum point occurs when $t = 2^{\frac{1}{3}}$.

At $t = 2^{\frac{1}{3}}$, $x = \dfrac{3t}{1+t^3} = \dfrac{3 \times 2^{\frac{1}{3}}}{1+2} = 2^{\frac{1}{3}}$

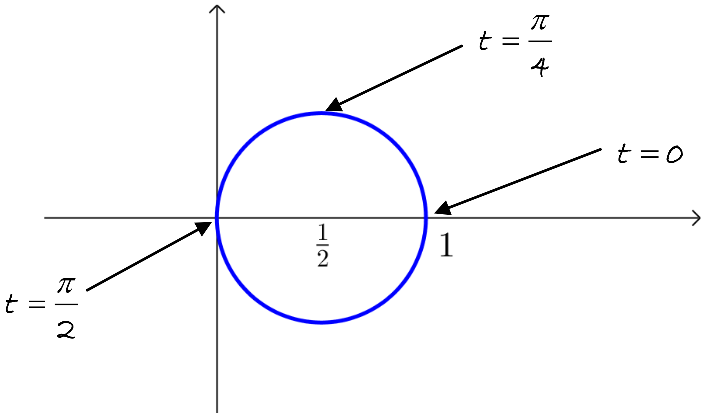
$$y = \frac{3t^2}{1+t^3} = \frac{3 \times 2^{\frac{2}{3}}}{1+2} = 2^{\frac{2}{3}}$$

so the maximum point is $\left(2^{\frac{1}{3}}, 2^{\frac{2}{3}}\right)$

3. (i) $y = \frac{1}{2}\sin 2t = \sin t \cos t$

$$y^2 = \sin^2 t \cos^2 t$$
$$= (1-\cos^2 t)\cos^2 t$$
$$= (1-x)x$$
$$= x - x^2$$
$$x^2 - x + y^2 = 0$$
$$(x - \tfrac{1}{2})^2 - \tfrac{1}{4} + y^2 = 0$$
$$(x - \tfrac{1}{2})^2 + y^2 = \tfrac{1}{4}$$

This is the equation of a circle, centre $(\frac{1}{2}, 0)$, radius $\frac{1}{2}$



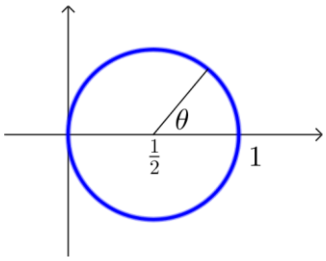
(ii) When $t = 0$, $x = 1$, $y = 0$

When $t = \dfrac{\pi}{4}$, $x = \frac{1}{2}$, $y = \frac{1}{2}$

When $t = \dfrac{\pi}{2}$, $x = 0$, $y = 0$

(iii) For $0 < t < \dfrac{\pi}{4}$, $\tan\theta = \dfrac{y}{x - \frac{1}{2}} = \dfrac{\frac{1}{2}\sin 2t}{\cos^2 t - \frac{1}{2}}$

$$= \frac{\sin 2t}{2\cos^2 t - 1}$$
$$= \frac{\sin 2t}{\cos 2t}$$
$$= \tan 2t$$



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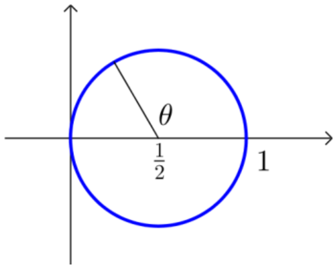

For $\frac{\pi}{4} < t < \frac{\pi}{2}$, $\tan\theta = \frac{y}{\frac{1}{2} - x} = \frac{\frac{1}{2}\sin 2t}{\frac{1}{2} - \cos^2 t}$

$$= \frac{\sin 2t}{1 - 2\cos^2 t}$$

$$= \frac{\sin 2t}{-\cos 2t}$$

$$= -\tan 2t$$