Section 1: Using parametric equations

Solutions to Exercise level 2

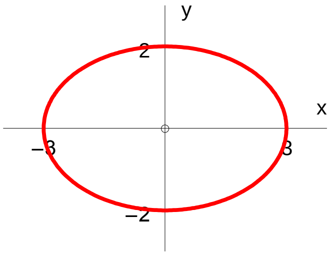
1. (i)

θ	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
x	3	2.9	2.6	2.1	1.5	0.8	0	-0.8	-1.5	-2.1	-2.6	-2.9	-3
y	0	0.5	1	1.4	1.7	1.9	2	1.9	1.7	1.4	1	0.5	0

(ii)

θ	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
x	-2.9	-2.6	-2.1	-1.5	-0.8	0	0.8	1.5	2.1	2.6	2.9	3
y	-0.5	-1	-1.4	-1.7	-1.9	-2	-1.9	-1.7	-1.4	-1	-0.5	0

(iii)



(iv) $x = 3\cos\theta \Rightarrow \frac{x}{3} = \cos\theta \Rightarrow \frac{x^2}{9} = \cos^2\theta$

$y = 2\sin\theta \Rightarrow \frac{y}{2} = \sin\theta \Rightarrow \frac{y^2}{4} = \sin^2\theta$

Adding: $\frac{x^2}{9} + \frac{y^2}{4} = \cos^2\theta + \sin^2\theta$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

2. (i) The curve is undefined for $t = 0$.

(ii) When $x = 0$, $t - \frac{1}{t} = 0 \Rightarrow t = \frac{1}{t} \Rightarrow t^2 = 1 \Rightarrow t = \pm 1$

When $t = 1$, $y = 2\left(1 + \frac{1}{1}\right) = 4$

When $t = -1$, $y = 2\left(-1 + \frac{1}{-1}\right) = -4$

so the curve crosses the y-axis at (0, 4) and (0, -4).

Edexcel A level Maths Parametric 1 Exercise solutions

When $y = 0$, $t + \frac{1}{t} = 0 \quad \Rightarrow t = -\frac{1}{t} \quad \Rightarrow t^2 = -1$

so the curve does not cross the x-axis.

(iii) $x = t - \frac{1}{t} \quad \Rightarrow 2x = 2t - \frac{2}{t}$

$y = 2\left(t + \frac{1}{t}\right) \quad \Rightarrow y = 2t + \frac{2}{t}$

Adding: $2x + y = 4t \quad \Rightarrow t = \frac{2x+y}{4}$

Substituting into $x = t - \frac{1}{t}$:

$$x = \frac{2x+y}{4} - \frac{4}{2x+y}$$

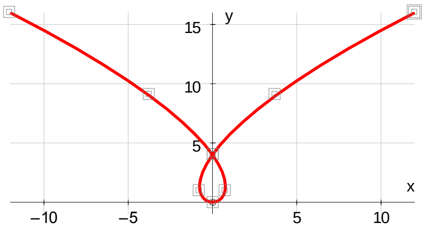
$$4x(2x+y) = (2x+y)^2 - 16$$

$$8x^2 + 4xy = 4x^2 + 4xy + y^2 - 16$$

$$y^2 = 4x^2 + 16$$

3.

t	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
x	-12	-3.75	0	0.75	0	-0.75	0	0.75	12
y	16	9	4	1	0	1	4	9	16



$x = 2t(t^2 - 1)$, $y = 4t^2$

$x^2 = 4t^2(t^2 - 1)^2$

$x^2 = y\left(\tfrac{1}{4}x - 1\right)^2$

Edexcel A level Maths Parametric 1 Exercise solutions

4. (i) $x = 16t\cos\theta \quad \Rightarrow t = \dfrac{x}{16\cos\theta}$

$$y = 16\sin\theta \times \frac{x}{16\cos\theta} - 5\left(\frac{x}{16\cos\theta}\right)^2$$

$$y = x\tan\theta - \frac{5x^2}{256}\sec^2\theta$$

$$y = x\tan\theta - \frac{5x^2}{256}(1+\tan^2\theta)$$

(ii) The ball bounces when $y = 0$.

$$16t\sin\theta - 5t^2 = 0$$

$$16t\sin 30° - 5t^2 = 0$$

$$8t - 5t^2 = 0$$

$$t(8-5t) = 0$$

$t = 0$ or $t = 1.6$

The ball bounces when $t = 1.6$

$x = 16 \times 1.6\cos 30° = 22.2$

so the horizontal distance travelled is 22.2 m (3 s.f.)

(iii) Maximum height occurs when $\dfrac{dy}{dt} = 0$.

$$\frac{dy}{dt} = 16\sin 30° - 10t = 8 - 10t$$

At maximum height, $8 - 10t = 0 \quad \Rightarrow t = 0.8$

When $t = 0.8$, $y = 8 \times 0.8 - 5 \times 0.8^2 = 3.2$

The maximum height is 3.2 m.