

Section 2: Further trigonometric equations

Solutions to Exercise level 2

1. $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$

$$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

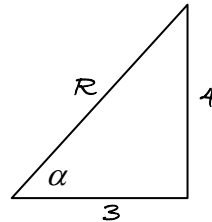
$$R\cos\alpha = 3$$

$$R\sin\alpha = 4$$

$$R^2 = 3^2 + 4^2 = 25 \Rightarrow R = 5$$

$$\tan\alpha = \frac{4}{3} \Rightarrow \alpha = 53.1^\circ$$

$$3\cos\theta + 4\sin\theta = 5\cos(\theta - 53.1^\circ)$$



The maximum value of the function is 5, and the minimum value is -5.

2. (i) $\cos\theta + \sin\theta = R\cos(\theta - \alpha)$

$$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

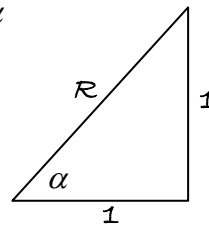
$$R\cos\alpha = 1$$

$$R\sin\alpha = 1$$

$$R^2 = 1^2 + 1^2 = 2 \Rightarrow R = \sqrt{2}$$

$$\tan\alpha = 1 \Rightarrow \alpha = 45^\circ$$

$$\cos\theta + \sin\theta = \sqrt{2}\cos(\theta - 45^\circ)$$



(ii) $2\cos\theta - \sin\theta = R\cos(\theta + \alpha)$

$$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$

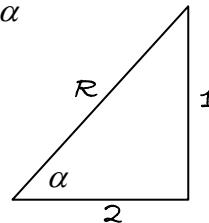
$$R\cos\alpha = 2$$

$$R\sin\alpha = 1$$

$$R^2 = 2^2 + 1^2 = 5 \Rightarrow R = \sqrt{5}$$

$$\tan\alpha = \frac{1}{2} \Rightarrow \alpha = 26.6^\circ$$

$$2\cos\theta - \sin\theta = \sqrt{5}\cos(\theta + 26.6^\circ)$$



(iii) $\sqrt{3}\sin\theta + \cos\theta = R\cos(\theta - \alpha)$

$$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

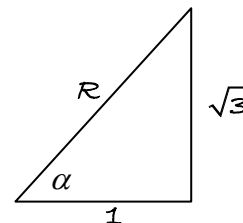
$$R\cos\alpha = 1$$

$$R\sin\alpha = \sqrt{3}$$

$$R^2 = \sqrt{3}^2 + 1^2 = 4 \Rightarrow R = 2$$

$$\tan\alpha = \sqrt{3} \Rightarrow \alpha = 60^\circ$$

$$\sqrt{3}\sin\theta + \cos\theta = 2\cos(\theta - 60^\circ)$$



Edexcel A level Maths Trig identities 2 Exercise solns

3. $5 \sin \theta - 8 \cos \theta = R \sin(\theta - \alpha)$

$$= R \sin \theta \cos \alpha - R \cos \theta \sin \alpha$$

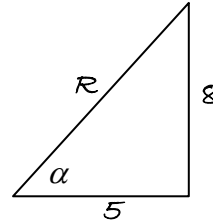
$$R \cos \alpha = 5$$

$$R \sin \alpha = 8$$

$$R^2 = 5^2 + 8^2 = 89 \Rightarrow R = \sqrt{89}$$

$$\tan \alpha = \frac{8}{5} \Rightarrow \alpha = 58.0^\circ$$

$$5 \sin \theta - 8 \cos \theta = \sqrt{89} \sin(\theta - 58.0^\circ)$$



$$5 \sin \theta - 8 \cos \theta = 6$$

$$\sqrt{89} \sin(\theta - 58.0^\circ) = 6$$

$$\sin(\theta - 58.0^\circ) = \frac{6}{\sqrt{89}}$$

$$\theta - 58.0^\circ = 39.5^\circ \text{ or } 140.5^\circ$$

$$\theta = 97.5^\circ \text{ or } 198.5^\circ$$

4. $7 \sin x + 24 \cos x = R \sin(x + \alpha)$

$$= R \sin x \cos \alpha + R \cos x \sin \alpha$$

$$R \cos \alpha = 7$$

$$R \sin \alpha = 24$$

$$R^2 = 7^2 + 24^2 = 625 \Rightarrow R = 25$$

$$\tan \alpha = \frac{24}{7} \Rightarrow \alpha = 1.29$$

$$7 \sin x + 24 \cos x = 25 \sin(x + 1.287)$$

Maximum value of function is 25.

This occurs when $x + 1.287 = \frac{\pi}{2} \Rightarrow x = 0.28$

Minimum value of function is -25.

This occurs when $x + 1.287 = \frac{3\pi}{2} \Rightarrow x = 3.43$

