

Section 2: Further trigonometric equations

Solutions to Exercise level 1

1. (i) $3 \sin(\theta + 20^\circ) = 2$

$$\sin(\theta + 20^\circ) = \frac{2}{3}$$

$$\theta + 20^\circ = 41.8^\circ \text{ or } 138.2^\circ$$

$$\theta = 21.8^\circ \text{ or } 118.2^\circ$$

(ii) $4 \cos(\theta + 50^\circ) = 1$

$$\cos(\theta + 50^\circ) = \frac{1}{4}$$

$$\theta + 50^\circ = 75.5^\circ - 284.5^\circ$$

$$\theta = 25.5^\circ \text{ or } 234.5^\circ$$

(iii) $7 \sin(\theta - 35^\circ) = 5$

$$\sin(\theta - 35^\circ) = \frac{5}{7}$$

$$\theta - 35^\circ = 45.6^\circ \text{ or } 134.4^\circ$$

$$\theta = 80.6^\circ \text{ or } 169.4^\circ$$

(iv) $5 \cos(\theta - 62^\circ) = 3$

$$\cos(\theta - 62^\circ) = \frac{3}{5}$$

$$\theta - 62^\circ = -53.1^\circ \text{ or } 53.1^\circ$$

$$\theta = 8.9^\circ \text{ or } 115.1^\circ$$

2. (i) $2 \sin \theta + 3 \cos \theta = R \sin(\theta + \alpha)$

$$= R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$$

Comparing coefficients of $\sin \theta$ gives $2 = R \cos \alpha$

Comparing coefficients of $\cos \theta$ gives $3 = R \sin \alpha$

(ii) $R^2 \cos^2 \alpha = 4$

$$R^2 \sin^2 \alpha = 9$$

$$R^2 = 13$$

$$R = \sqrt{13}$$

$$\frac{R \sin \alpha}{R \cos \alpha} = \frac{3}{2}$$

$$\tan \alpha = \frac{3}{2}$$

$$\alpha = 56.3^\circ \text{ (1 d.p.)}$$

3. (i) $\sin \theta - 4 \cos \theta = R \sin(\theta - \alpha)$

$$= R \sin \theta \cos \alpha - R \cos \theta \sin \alpha$$

Comparing coefficients of $\sin \theta$ gives $1 = R \cos \alpha$

Comparing coefficients of $\cos \theta$ gives $4 = R \sin \alpha$

Edexcel A level Maths Trig identities 2 Exercise solns

$$\begin{aligned} \text{(ii)} \quad R^2 \cos^2 \alpha &= 1 & \frac{R \sin \alpha}{R \cos \alpha} &= \frac{4}{1} \\ R^2 \sin^2 \alpha &= 16 & \tan \alpha &= 4 \\ R^2 &= 17 & \alpha &= 76.0^\circ \text{ (1 d.p.)} \\ R &= \sqrt{17} \end{aligned}$$

$$\begin{aligned} 4. \text{ (i)} \quad 4 \sin \theta - 5 \cos \theta &= R \cos(\theta + \alpha) \\ &= R \cos \theta \cos \alpha - R \sin \theta \sin \alpha \\ \text{Comparing coefficients of } \cos \theta &\text{ gives } 4 = R \cos \alpha \\ \text{Comparing coefficients of } \sin \theta &\text{ gives } 5 = R \sin \alpha \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad R^2 \cos^2 \alpha &= 16 & \frac{R \sin \alpha}{R \cos \alpha} &= \frac{5}{4} \\ R^2 \sin^2 \alpha &= 25 & \tan \alpha &= 1.25 \\ R^2 &= 41 & \alpha &= 51.3^\circ \text{ (1 d.p.)} \\ R &= \sqrt{41} \end{aligned}$$

$$\begin{aligned} 5. \text{ (i)} \quad 3 \sin \theta + \cos \theta &= R \cos(\theta - \alpha) \\ &= R \cos \theta \cos \alpha + R \sin \theta \sin \alpha \\ \text{Comparing coefficients of } \cos \theta &\text{ gives } 1 = R \cos \alpha \\ \text{Comparing coefficients of } \sin \theta &\text{ gives } 3 = R \sin \alpha \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad R^2 \cos^2 \alpha &= 1 & \frac{R \sin \alpha}{R \cos \alpha} &= \frac{3}{1} \\ R^2 \sin^2 \alpha &= 9 & \tan \alpha &= 3 \\ R^2 &= 10 & \alpha &= 71.6^\circ \text{ (1 d.p.)} \\ R &= \sqrt{10} \end{aligned}$$