Section 1: The compound angle identities

Solutions to Exercise level 3

1. $$\left[1-\cos\left(\theta+\frac{\pi}{4}\right)\right]\left[1+\sin\left(\theta+\frac{\pi}{4}\right)\right]$$

$$=\left(1-\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right)\left(1+\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right)$$

$$=\left(1+\frac{1}{\sqrt{2}}\sin\theta\right)^2-\frac{1}{2}\cos^2\theta$$

$$=1+\frac{2}{\sqrt{2}}\sin\theta+\frac{1}{2}\sin^2\theta-\frac{1}{2}(1-\sin^2\theta)$$

$$=\frac{1}{2}+\frac{2}{\sqrt{2}}\sin\theta+\sin^2\theta$$

$$=\left(\frac{1}{\sqrt{2}+\sin\theta}\right)^2$$

$$\left(\frac{1}{\sqrt{2}}+\sin\theta\right)^2=k$$

$$\frac{1}{\sqrt{2}}+\sin\theta=\sqrt{k}$$

$$\sin\theta=\sqrt{k}-\frac{1}{\sqrt{2}}$$

$$-1\le\sqrt{k}-\frac{1}{\sqrt{2}}\le 1$$

$$-1+\frac{1}{\sqrt{2}}\le\sqrt{k}\le 1+\frac{1}{\sqrt{2}}$$

$$-1+\frac{1}{\sqrt{2}}<0,\text{ so }\sqrt{k}\le 1+\frac{1}{\sqrt{2}}$$

$$k\le\left(1+\frac{1}{\sqrt{2}}\right)^2=1+\frac{2}{\sqrt{2}}+\frac{1}{2}=\frac{3}{2}+\sqrt{2}$$

So there are no real roots if $k>\frac{3}{2}+\sqrt{2}$ or $k<0$

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2. $\tan 3\theta = \tan(2\theta + \theta) = \dfrac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta}$

$$= \frac{\frac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1 - \tan\theta \times \frac{2\tan\theta}{1-\tan^2\theta}}$$

$$= \frac{2\tan\theta + \tan\theta(1-\tan^2\theta)}{(1-\tan^2\theta) - \tan\theta \times 2\tan\theta}$$

$$= \frac{2\tan\theta + \tan\theta - \tan^3\theta}{1 - \tan^2\theta - 2\tan^2\theta}$$

$$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = 1$$

$3\tan\theta - \tan^3\theta = 1 - 3\tan^2\theta$

$\tan^3\theta - 3\tan^2\theta - 3\tan\theta + 1 = 0$

$(\tan\theta + 1)(\tan^2\theta - 4\tan\theta + 1) = 0$

$\tan\theta = -1 \quad \Rightarrow \theta = \dfrac{3\pi}{4}$ which is not in the required range

so $\tan^2\theta - 4\tan\theta + 1 = 0$

$$\tan\theta = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$$

$\tan\theta = 2 + \sqrt{3} \quad \Rightarrow \theta = 1.31$ which is not in the required range

$\tan\theta = 2 - \sqrt{3} \quad \Rightarrow \theta = 0.262$

so $\tan\theta = 2 - \sqrt{3}$

3. (i) $\sin 3x = \sin(2x + x)$

$= \sin 2x \cos x + \cos 2x \sin x$

$= 2\sin x \cos x \cos x + (2\cos^2 x - 1)\sin x$

$= \sin x(2\cos^2 x + 2\cos^2 x - 1)$

$= \sin x(4\cos^2 x - 1)$

(ii) $\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x = 0$

$\sin x - \frac{1}{2} \times 2\sin x \cos x + \frac{1}{3}\sin x(4\cos^2 x - 1) = 0$

$\frac{1}{3}\sin x(3 - 3\cos x + 4\cos^2 x - 1) = 0$

$\frac{1}{3}\sin x(4\cos^2 x - 3\cos x + 2) = 0$

$\sin x \neq 0$ in the interval $0 < x < \pi$

The discriminant of the equation $4\cos^2 x - 3\cos x + 2 = 0$ is

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$9-4\times4\times2$. This is negative so the equation has no real roots.

So there are no values of x in the interval $0<x<\pi$ for which $f(x)=0$.