Section 1: The compound angle identities

Solutions to Exercise level 2

1. $\cos A\cos(A-B)+\sin A\sin(A-B)=\cos(A-(A-B))$
$=\cos B$

2. (i) $1-2\sin x-4\cos 2x=0$
$1-2\sin x-4(1-2\sin^2 x)=0$
$1-2\sin x-4+8\sin^2 x=0$
$8\sin^2 x-2\sin x-3=0$
$(4\sin x-3)(2\sin x+1)=0$
$\sin x=\frac{3}{4}$ or $-\frac{1}{2}$
$\sin x=\frac{3}{4} \Rightarrow x=48.6°$ or $131.4°$
$\sin x=-\frac{1}{2} \Rightarrow x=210°$ or $330°$
$x=48.6°,131.4°,210°,330°$

sin is positive in the first and second quadrants

sin is negative in the third and fourth quadrants

(ii) $\sin 2x-\tan x=0$
$2\sin x\cos x-\frac{\sin x}{\cos x}=0$
$2\sin x\cos^2 x-\sin x=0$
$\sin x(2\cos^2 x-1)=0$
$\sin x=0$ or $\cos x=\pm\frac{1}{\sqrt{2}}$
$\sin x=0 \Rightarrow x=0°$ or $180°$ or $360°$
$\cos x=\frac{1}{\sqrt{2}} \Rightarrow x=45°$ or $225°$
$\cos x=-\frac{1}{\sqrt{2}} \Rightarrow x=135°$ or $315°$
$x=0°,45°,135°,180°,225°,315°,360°$

$\cos 2A=1-2\sin^2 A$

(iii) $\tan 2x+\tan x=0$
$\frac{2\tan x}{1-\tan^2 x}+\tan x=0$
$2\tan x+\tan x(1-\tan^2 x)=0$
$\tan x(2+1-\tan^2 x)=0$
$\tan x(3-\tan^2 x)=0$
$\tan x(\tan x+\sqrt{3})(\tan x-\sqrt{3})=0$
$\tan x=0$ or $-\sqrt{3}$ or $\sqrt{3}$
$\tan x=0 \Rightarrow x=0°$ or $180°$ or $360°$

Edexcel A level Maths Trig identities 1 Exercise soln

$\tan x = -\sqrt{3} \Rightarrow x = 120° \text{ or } 240°$

$\tan x = \sqrt{3} \Rightarrow x = 60° \text{ or } 300°$

The values of x are $0°, 60°, 120°, 180°, 240°, 300°, 360°$

3. (i) L.H.S. $= \cos^4 x - \sin^4 x$

Difference of two squares

$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$

$= 1 \times \cos 2x$

$= \cos 2x$

$= \text{R.H.S}$

$\cos^2 A + \sin^2 A = 1$ and
$\cos 2A = \cos^2 A - \sin^2 A$

(ii) L.H.S. $= \dfrac{\cos x - \sin x}{\cos x + \sin x}$

$= \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)(\cos x + \sin x)}$

$= \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + 2\sin x \cos x + \sin^2 x}$

$= \dfrac{\cos 2x}{1 + \sin 2x}$

$= \text{R.H.S.}$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos^2 A + \sin^2 A = 1$ and
$\sin 2A = 2\sin A \cos A$

(iii) L.H.S. $= \dfrac{\cot^2 x - 1}{\cot^2 x + 1} = \dfrac{\frac{\cos^2 x}{\sin^2 x} - 1}{\frac{\cos^2 x}{\sin^2 x} + 1}$

$= \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$

$= \dfrac{\cos 2x}{1}$

$= \cos 2x$

$= \text{R.H.S.}$

(iv) L.H.S. $= \dfrac{\sin x}{1 + \cos x} + \dfrac{1 - \cos x}{\sin x}$

$= \dfrac{\sin^2 x + (1 - \cos x)(1 + \cos x)}{(1 + \cos x)\sin x}$

$= \dfrac{\sin^2 x + 1 - \cos^2 x}{(1 + \cos x)\sin x}$

$= \dfrac{2\sin^2 x}{(1 + \cos x)\sin x}$

$= \dfrac{2\sin x}{1 + \cos x}$

$= \dfrac{4\sin \frac{1}{2}x \cos \frac{1}{2}x}{2\cos^2 \frac{1}{2}x}$

$\cos^2 A + \sin^2 A = 1$

$\sin 2A = 2\sin A \cos A$ and
$\cos 2A = \cos^2 A - \sin^2 A$

$$= \frac{2\sin\frac{1}{2}x}{\cos\frac{1}{2}x}$$
$$= 2\tan\frac{1}{2}x$$
$$= \text{R.H.S.}$$

4. $\sin 3x = \sin x$

$\sin(2x + x) = \sin x$

$\sin 2x \cos x + \cos 2x \sin x = \sin x$

$2\sin x \cos x \cos x + (2\cos^2 x - 1)\sin x = \sin x$

$2\sin x \cos^2 x + 2\sin x \cos^2 x - \sin x = \sin x$

$4\sin x \cos^2 x - 2\sin x = 0$

$\sin x(2\cos^2 x - 1) = 0$

$\sin x = 0 \quad \text{or} \quad \cos x = \pm\frac{1}{\sqrt{2}}$

$\sin x = 0 \quad \Rightarrow x = 0° \text{ or } 180°$

$\cos x = \pm\frac{1}{\sqrt{2}} \quad \Rightarrow x = 45° \text{ or } 135°$

The solutions are 0°, 45°, 135°, 180°.