

Section 1: The compound angle identities

Solutions to Exercise level 1

1. $\cos x = \frac{3}{5}$

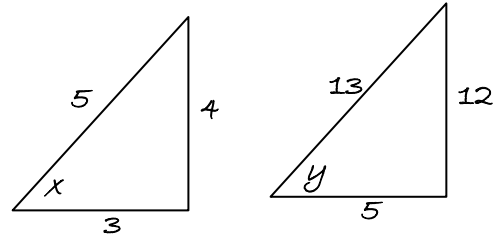
$$\sin x = \sqrt{1 - \cos^2 x} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\sin 2x = 2 \sin x \cos x = 2 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{25}$$

$$\cos 2x = 2 \cos^2 x - 1 = 2 \times \left(\frac{3}{5}\right)^2 - 1 = \frac{18}{25} - 1 = -\frac{7}{25}$$

$$\begin{aligned} \sin 3x &= \sin(2x + x) \\ &= \sin 2x \cos x + \cos 2x \sin x \\ &= \frac{24}{25} \times \frac{3}{5} + \left(-\frac{7}{25}\right) \times \frac{4}{5} \\ &= \frac{72 - 28}{125} \\ &= \frac{44}{125} \end{aligned}$$

2. $\cos x = \frac{3}{5} \Rightarrow \sin x = \frac{4}{5}$ and $\tan x = \frac{4}{3}$
 $\sin y = \frac{12}{13} \Rightarrow \cos y = \frac{5}{13}$ and $\tan y = \frac{12}{5}$



$$\begin{aligned} \sin(x + y) &= \sin x \cos y + \cos x \sin y \\ &= \frac{4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{12}{13} \\ &= \frac{20 + 36}{65} = \frac{56}{65} \end{aligned}$$

$$\begin{aligned} \cos(x - y) &= \cos x \cos y + \sin x \sin y \\ &= \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13} \\ &= \frac{15 + 48}{65} = \frac{63}{65} \end{aligned}$$

$$\begin{aligned} \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x} \\ &= \frac{2 \times \frac{4}{3}}{1 - \left(\frac{4}{3}\right)^2} \\ &= \frac{\frac{8}{3}}{1 - \frac{16}{9}} \\ &= \frac{24}{9 - 16} = -\frac{24}{7} \end{aligned}$$

Edexcel A level Maths Trig identities 1 Exercise solns

3. (i) $\cos 3x \cos 2x - \sin 3x \sin 2x = 0.5$

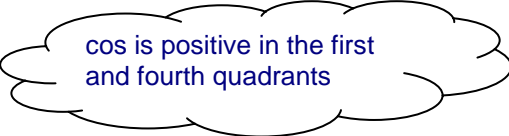
$$\cos(3x + 2x) = 0.5$$

$$\cos 5x = 0.5$$

$$5x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}$$

$$x = \frac{\pi}{15}, \frac{\pi}{3}, \frac{7\pi}{15}, \frac{11\pi}{15}, \frac{13\pi}{15}$$


$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$



cos is positive in the first and fourth quadrants

(ii) $\sin 5x \cos x + \cos 5x \sin x = 1$

$$\sin(5x + x) = 1$$

$$\sin 6x = 1$$

$$6x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$$