Section 1: Trigonometric functions and identities

Solutions to Exercise level 2

1. (i) $\dfrac{1-\sec^2 x}{1-\operatorname{cosec}^2 x} = \dfrac{-\tan^2 x}{-\cot^2 x}$

$= \tan^4 x$

(ii) $\dfrac{1-\sin^2 x}{1+\tan^2 x} = \dfrac{\cos^2 x}{\sec^2 x}$

$= \cos^4 x$

(iii) $\dfrac{\sin x}{1+\cot^2 x} = \dfrac{\sin x}{\operatorname{cosec}^2 x}$

$= \sin^3 x$

2. (i) $\operatorname{cosec}(x+10°) = 3$

$\sin(x+10°) = \frac{1}{3}$

$x+10° = 19.5°$ or $160.5°$

$x = 9.5°$ or $150.5°$ (1 d.p.)

sin is positive in the 1st and 2nd quadrants

(ii) $\cot(x-30°) = 0.45$

$\tan(x-30°) = \dfrac{1}{0.45}$

$x-30° = 65.8°$

$x = 95.8°$ (3 s.f.)

There is only one solution in the required range

3. (i) (a) $f(x) = 1+\sin 2x$ for $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$

The range of the function is $0 \le f(x) \le 2$

(b) $y = 1+\sin 2x$

Interchanging x and y: $x = 1+\sin 2y$

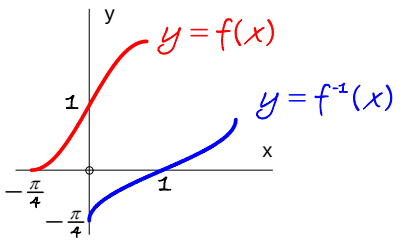
$x-1 = \sin 2y$

$\arcsin(x-1) = 2y$

$y = \frac{1}{2}\arcsin(x-1)$

Inverse function is $f^{-1}(x) = \frac{1}{2}\arcsin(x-1)$, for $0 \le x \le 2$.

(c)



(ii) (a) $f(x) = \frac{1}{2}\tan(x - \frac{\pi}{2})$ for $0 \le x \le \pi$

The range of the function is $f(x) \in \mathbb{R}$

(b) $y = \frac{1}{2}\tan(x - \frac{\pi}{2})$

Interchanging x and y: $x = \frac{1}{2}\tan(y - \frac{\pi}{2})$

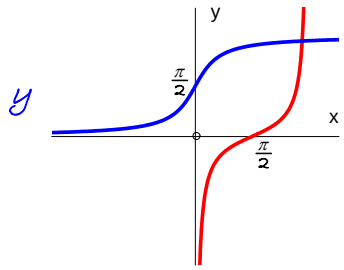
$2x = \tan(y - \frac{\pi}{2})$

$\arctan 2x = y - \frac{\pi}{2}$

$y = \frac{\pi}{2} + \arctan 2x$

Inverse function is $f^{-1}(x) = \frac{\pi}{2} + \arctan 2x$, for $x \in \mathbb{R}$

(c)



4. (i) $\text{LHS} \equiv \tan^2\theta + \cot^2\theta$

$\equiv \sec^2\theta - 1 + \operatorname{cosec}^2\theta - 1$

$\equiv \sec^2\theta + \operatorname{cosec}^2\theta - 2$

$\equiv \text{RHS}$

(ii) $\text{RHS} \equiv \dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta}$

$\equiv \dfrac{\operatorname{cosec}\theta \sin\theta}{\sin\theta(\operatorname{cosec}\theta - \sin\theta)}$ Multiplying top and bottom by $\sin\theta$

$\equiv \dfrac{1}{1 - \sin^2\theta}$ since $\sin\theta \operatorname{cosec}\theta = 1$

$\equiv \dfrac{1}{\cos^2\theta}$

$\equiv \sec^2\theta \equiv \text{LHS}$

(iii) $\text{LHS} \equiv (1-\cos\theta)(1+\sec\theta)$

$\equiv 1-\cos\theta+\sec\theta-\cos\theta\sec\theta$

$\equiv 1-\cos\theta+\sec\theta-1$

$\equiv \sec\theta-\cos\theta$

$\equiv \frac{1}{\cos\theta}-\cos\theta$

$\equiv \frac{1-\cos^2\theta}{\cos\theta}$

$\equiv \frac{\sin^2\theta}{\cos\theta}$

$= \sin\theta\times\frac{\sin\theta}{\cos\theta}$

$\equiv \sin\theta\tan\theta \equiv \text{RHS}$

5. (i) $2\tan x\operatorname{cosec} x = 3$

$2\frac{\cancel{\sin x}}{\cos x}\times\frac{1}{\cancel{\sin x}} = 3$

$2 = 3\cos x$

$\cos x = \frac{2}{3}$

$x = 48.2^\circ$ or 311.8°

cos is positive in the 1st and 4th quadrants

(ii) $\tan^2 x = \sec x\tan x$

$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos x}\times\frac{\sin x}{\cos x}$

$\sin^2 x = \sin x$ for $\cos x \neq 0$

$\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$

$\sin x = 0$ or 1, $\cos x \neq 0$

$x = 0^\circ$ or 180° or 360°

6. (i) $2\tan x - \cot x = 1$

$2\tan x - \frac{1}{\tan x} = 1$

$2\tan^2 x - 1 = \tan x$

$2\tan^2 x - \tan x - 1 = 0$

$(2\tan x + 1)(\tan x - 1) = 0$

$\tan x = -\frac{1}{2}$ or 1

$\tan x = -\frac{1}{2} \Rightarrow x = -26.6^\circ$ or 153.4 (1 d.p.)

$\tan x = 1 \Rightarrow x = -135^\circ$ or 45°

$x = -135°, -26.6°, 45°$ or 153.4

(ii) $2\cos x = 3 - \sec x$

$$2\cos x = 3 - \frac{1}{\cos x}$$

$2\cos^2 x = 3\cos x - 1$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = \frac{1}{2}$ or 1

$\cos x = \frac{1}{2} \Rightarrow x = \pm 60°$

$\cos x = 1 \Rightarrow x = 0°$.

$x = -60°, 0°$ or $60°$

(iii) $\tan^2 x = \sec x + 5$

$\sec^2 x - 1 = \sec x + 5$

$\sec^2 x - \sec x - 6 = 0$

$(\sec x - 3)(\sec x + 2) = 0$

$\sec x = 3$ or -2

$\cos x = \frac{1}{3}$ or $-\frac{1}{2}$

$\cos x = \frac{1}{3} \Rightarrow x = \pm 70.5°$ (1 d.p.)

$\cos x = -\frac{1}{2} \Rightarrow x = \pm 120°$

$x = -120°, -70.5°, 70.5°, 120°$