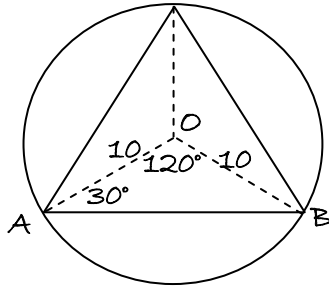


Section 2: Circular measure

Solutions to Exercise level 2

1.

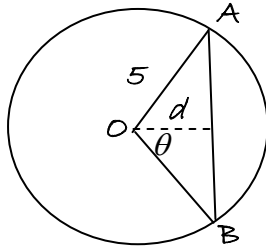


(i) Area of circle = $\pi \times 10^2 = 314.16 \text{ cm}^2$

(ii) Area of triangle AOB = $\frac{1}{2} \times 10 \times 10 \sin 120^\circ = 43.30$
 Total area of triangle = $3 \times 43.30 = 129.90 \text{ cm}^2$

(iii) Area of segments = $314.16 - 129.9 = 184.26 \text{ cm}^2$

2.



Arc length = $r\theta$

$9.3 = 5 \times 2\theta$

$\theta = 0.93$

$d = 5 \cos 0.93 = 2.99 \text{ cm}$

3. (i) $3 \cos x - \sin 2x = 2$
 using small angle approximations:

$$3\left(1 - \frac{1}{2}x^2\right) - 2x = 2$$

$$3 - \frac{3}{2}x^2 - 2x = 2$$

$$3x^2 + 4x - 2 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 4 \times 3 \times -2}}{6} = \frac{-4 \pm \sqrt{40}}{6}$$

Smaller root is 0.39

So the smallest positive root is approximately 0.39.

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(ii) $x \tan x = 5 \sin x - \cos x$

Using small angle approximations:

$$x \times x = 5x - (1 - \frac{1}{2}x^2)$$

$$x^2 = 5x - 1 + \frac{1}{2}x^2$$

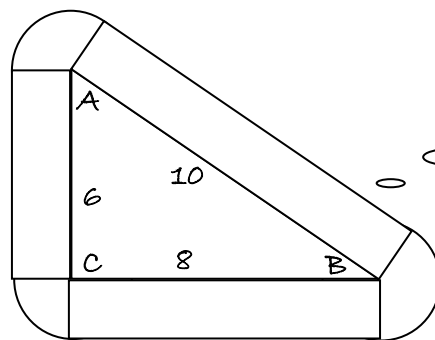
$$x^2 - 10x + 2 = 0$$

$$x = \frac{10 \pm \sqrt{100 - 4 \times 2 \times 1}}{2} = \frac{10 \pm \sqrt{92}}{2}$$

Smaller root is 0.204

So the smallest positive root is approximately 0.204.

4.



Since $6^2 + 8^2 = 10^2$, the triangle is right-angled.

Angle at A is given by $\sin A = \frac{8}{10} \Rightarrow A = 0.927$

Angle in sector of circle at A = $\pi - 0.927 = 2.214$

Area of sector of circle at A = $\frac{1}{2} \times 1^2 \times 2.214 = 1.107 \text{ km}^2$

Angle at B is given by $\sin B = \frac{6}{10} \Rightarrow B = 0.6435$

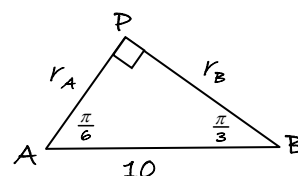
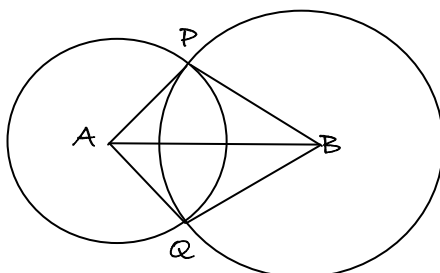
Angle in sector of circle at B = $\pi - 0.6435 = 2.498$

Area of sector of circle at B = $\frac{1}{2} \times 1^2 \times 2.498 = 1.249 \text{ km}^2$

Area of quarter circle at C = $\frac{1}{2} \times \pi \times 1^2 = 0.785$

Total area of exclusion zone = $6 + 8 + 10 + 1.107 + 1.249 + 0.785$
 $= 27.14 \text{ km}^2$

5.



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$$r_A = 10 \cos \frac{\pi}{6} = 10 \times \frac{1}{2} \sqrt{3} = 5\sqrt{3}$$

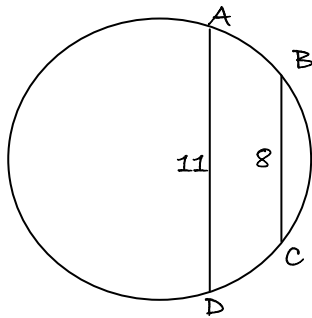
$$\text{Arc length PQ for circle with centre A} = r_A \times \frac{\pi}{3} = \frac{5\sqrt{3}\pi}{3} = 9.07 \text{ cm}$$

$$r_B = 10 \cos \frac{\pi}{3} = 10 \times \frac{1}{2} = 5$$

$$\text{Arc length PQ for circle with centre B} = r_B \times \frac{2\pi}{3} = 5 \times \frac{2\pi}{3} = 10.47 \text{ cm}$$

$$\text{Total perimeter} = 9.07 + 10.47 \text{ cm} = 19.54 \text{ cm}$$

6.



Let angle subtended by chord AD be α

$$2 \times 6 \sin \frac{1}{2} \alpha = 11 \Rightarrow \alpha = 2.319$$

Let angle subtended by chord BC be β

$$2 \times 6 \sin \frac{1}{2} \beta = 8 \Rightarrow \beta = 1.459$$

Length of arc AD = 6α

Length of arc BC = 6β

Length of arcs AB and CD = $6(\alpha - \beta) = 5.16$

Perimeter of region = $11 + 8 + 5.16 = 24.16 \text{ cm}$

Area of sector OAD = $\frac{1}{2} \times 6^2 \alpha = 18\alpha$

Area of triangle OAD = $\frac{1}{2} \times 6^2 \sin \alpha = 18 \sin \alpha$

Area of minor segment bounded by AD = $18(\alpha - \sin \alpha)$

Similarly, area of minor segment bounded by BC = $18(\beta - \sin \beta)$

Area of region ABCD = $18(\alpha - \sin \alpha) - 18(\beta - \sin \beta) = 20.18 \text{ cm}^2$

Edexcel A level Maths Trigonometry 2 Exercise solns

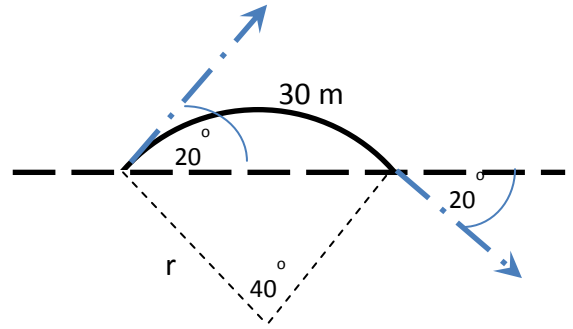
7. (i) The arc subtends 40° at the centre.

$$\begin{aligned}40^\circ &= 40 \left(\frac{\pi}{180} \right) \\ &= \frac{2\pi}{9} \quad (\approx 0.6981)\end{aligned}$$

$$\Rightarrow 30 = r \left(\frac{2\pi}{9} \right)$$

$$\Rightarrow r = \frac{270}{2\pi} \quad (\approx 42.97)$$

so the roadway curve radius is $\frac{270}{2\pi}$ m.



- (ii) Area of brickwork = area of sector - area of triangle

$$\begin{aligned}&= \frac{1}{2} \left(\frac{270}{2\pi} \right)^2 \left(\frac{2\pi}{9} \right) - 2 \left(\frac{1}{2} \right) \left(r \sin \frac{\pi}{9} \right) \left(r \cos \frac{\pi}{9} \right) \\ &\approx 51 \text{ m}^2\end{aligned}$$