Edexcel A level Mathematics Further algebra

Section 1: The general binomial expansion

Solutions to Exercise level 3 (Extension)

1. $$(1+ax)^n = 1 + nax + \frac{n(n-1)}{1\times 2}(ax)^2 + \frac{n(n-1)(n-2)}{1\times 2}(ax)^3 + \ldots$$

Comparing with $1 - x + \frac{1\times 3}{1\times 2}x^2 - \frac{1\times 3\times 5}{1\times 2\times 4}x^3 + \ldots$

Comparing terms in x gives $na = -1$

Comparing terms in x^2 gives $\frac{n(n-1)}{1\times 2}a^2 = \frac{1\times 3}{1\times 2}$

$$n(n-1)a^2 = 3$$
$$n^2a^2 - na^2 = 3$$
$$(-1)^2 - (-1)a = 3$$
$$1 + a = 3$$
$$a = 2,\ n = -\tfrac{1}{2}$$

$$(1+2x)^{-\frac{1}{2}} = 1 - \tfrac{1}{2}\times 2x + \frac{-\frac{1}{2}\times -\frac{3}{2}}{1\times 2}(2x)^2 + \frac{-\frac{1}{2}\times -\frac{3}{2}\times -\frac{5}{2}}{1\times 2}(2x)^3 + \ldots$$
$$= 1 - x + \frac{1\times 3}{1\times 2}x^2 - \frac{1\times 3\times 5}{1\times 2\times 3}x^2 + \ldots$$

so $1 - x + \frac{1\times 3}{1\times 2}x^2 - \frac{1\times 3\times 5}{1\times 2\times 4}x^3 + \ldots = \frac{1}{\sqrt{1+2x}}$

2. $$\frac{1}{1-x} - \frac{1}{1+x} = (1-x)^{-1} - (1+x)^{-1}$$
$$= 1 - (-x) + \frac{-1\times -2}{1\times 2}(-x)^2 + \ldots - \left(1 - x + \frac{-1\times -2}{1\times 2}x^2 + \ldots\right)$$
$$= 1 + x + x^2 + \ldots - 1 + x - x^2 + \ldots$$
$$\approx 2x$$

Putting $x = 0.001$, $\frac{1}{1-0.001} - \frac{1}{1+0.001} \approx 0.002$

$$\frac{1}{0.999} - \frac{1}{1.001} \approx 0.002$$
$$\frac{1000}{0.999} - \frac{1000}{1.001} \approx 2$$

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3. $\left(1+\sqrt{2}x\right)^{-2} = 1-2\sqrt{2}x+\frac{-2\times-3}{1\times2}\left(\sqrt{2}x\right)^2+\frac{-2\times-3\times-4}{1\times2\times3}\left(\sqrt{2}x\right)^2+\ldots$

$$= 1-2\sqrt{2}x+3\left(\sqrt{2}x\right)^2-4\left(\sqrt{2}x\right)^3+\ldots+(-1)^r\times(r+1)\left(\sqrt{2}x\right)^r+\ldots$$

When $r=2n$, coefficient $=(-1)^{2n}\times(2n+1)\left(\sqrt{2}\right)^{2n}$

$$=(2n+1)2^n$$

4. $(2+x)^{-2} = x^{-2}\left(\frac{2}{x}+1\right)^{-2}$

$$= \frac{1}{x^2}\left(1+\frac{2}{x}\right)^{-2}$$

$$= \frac{1}{x^2}\left(1-2\times\frac{2}{x}+\frac{-2\times-3}{1\times2}\left(\frac{2}{x}\right)^2+\frac{-2\times-3\times-4}{1\times2\times3}\left(\frac{2}{x}\right)^3+\ldots\right)^{-2}$$

$$= \frac{1}{x^2}\left(1-\frac{1}{x}+12\left(\frac{1}{x}\right)^2-32\left(\frac{1}{x}\right)^3+\ldots\right)$$

$$= \frac{1}{x^2}-\frac{1}{x^3}+\frac{12}{x^4}-\frac{32}{x^5}+\ldots$$

Valid for $\left|\frac{2}{x}\right|<1$

$$\left|\frac{x}{2}\right|>1$$

$$|x|>2$$

5. $(1+nx)^n = 1+n(nx)+\frac{n(n-1)}{1\times2}(nx)^2+\frac{n(n-1)(n-2)}{1\times2\times3}(nx)^3+\ldots$

Coefficients of x^2 and x^3 are equal,

so $\frac{n(n-1)}{1\times2}n^2 = \frac{n(n-1)(n-2)}{1\times2\times3}n^3$

$$n^4(n-1)(n-2)-3n^3(n-1)=0$$

$$n^3(n-1)(n(n-2)-3)=0$$

$$n^3(n-1)(n^2-2n-3)=0$$

$$n^3(n-1)(n+1)(n-3)=0$$

The possible values of n are $0, 1, -1$ and 3.