Section 1: The general binomial expansion

Solutions to Exercise level 2

1. (i) $\dfrac{1}{3-x} = (3-x)^{-1} = 3^{-1}(1-\frac{1}{3}x)^{-1} = \frac{1}{3}(1-\frac{1}{3}x)^{-1}$

$$= \tfrac{1}{3}\left(1 + (-1)(-\tfrac{1}{3}x) + \frac{-1\times-2}{1\times2}(-\tfrac{1}{3}x)^2 + \frac{-1\times-2\times-3}{1\times2\times3}(-\tfrac{1}{3}x)^3 + \ldots\right)$$

$$= \tfrac{1}{3}\left(1 + \tfrac{1}{3}x + \tfrac{1}{9}x^2 + \tfrac{1}{27}x^3 + \ldots\right)$$

$$= \tfrac{1}{3} + \tfrac{1}{9}x + \tfrac{1}{27}x^2 + \tfrac{1}{81}x^3 + \ldots$$

Valid for $-1 < \frac{1}{3}x < 1$

$\Rightarrow -3 < x < 3$

(ii) $\sqrt{4-3x} = (4-3x)^{\frac{1}{2}} = 4^{\frac{1}{2}}(1-\frac{3}{4}x)^{\frac{1}{2}} = 2(1-\frac{3}{4}x)^{\frac{1}{2}}$

$$= 2\left(1 + \tfrac{1}{2}(-\tfrac{3}{4}x) + \frac{\frac{1}{2}\times-\frac{1}{2}}{1\times2}(-\tfrac{3}{4}x)^2 + \frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{1\times2\times3}(-\tfrac{3}{4}x)^3 + \ldots\right)$$

$$= 2\left(1 - \tfrac{3}{8}x - \tfrac{9}{128}x^2 - \tfrac{27}{1024} + \ldots\right)$$

$$= 2 - \tfrac{3}{4}x - \tfrac{9}{64}x^2 - \tfrac{27}{512} + \ldots$$

Valid for $-1 < \frac{3}{4}x < 1$

$\Rightarrow -\frac{4}{3} < x < \frac{4}{3}$

(iii) $\dfrac{2+x}{2-x} = (2+x)(2-x)^{-1} = (2+x)\times2^{-1}(1-\frac{1}{2}x)^{-1} = \frac{1}{2}(2+x)(1-\frac{1}{2}x)^{-1}$

$$= \tfrac{1}{2}(2+x)\left(1 + (-1)(-\tfrac{1}{2}x) + \frac{-1\times-2}{1\times2}(-\tfrac{1}{2}x)^2 + \frac{-1\times-2\times-3}{1\times2\times3}(-\tfrac{1}{2}x)^3 + \ldots\right)$$

$$= \tfrac{1}{2}(2+x)\left(1 + \tfrac{1}{2}x + \tfrac{1}{4}x^2 + \tfrac{1}{8}x^3 + \ldots\right)$$

$$= \tfrac{1}{2}(2 + x + \tfrac{1}{2}x^2 + \tfrac{1}{4}x^3 + x + \tfrac{1}{2}x^2 + \tfrac{1}{4}x^3 + \ldots)$$

$$= \tfrac{1}{2}(2 + 2x + x^2 + \tfrac{1}{2}x^3 + \ldots)$$

$$= 1 + x + \tfrac{1}{2}x^2 + \tfrac{1}{4}x^3 + \ldots$$

Valid for $-1 < \frac{1}{2}x < 1$

$\Rightarrow -2 < x < 2$

(iv) $\sqrt{\dfrac{1-x}{1-2x}} = (1-x)^{\frac{1}{2}}(1-2x)^{-\frac{1}{2}}$

$$(1-x)^{\frac{1}{2}} = 1 + \tfrac{1}{2}(-x) + \frac{\frac{1}{2}\times-\frac{1}{2}}{1\times2}(-x)^2 + \frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{1\times2\times3}(-x)^3 + \ldots$$

$$= 1 - \tfrac{1}{2}x - \tfrac{1}{8}x^2 - \tfrac{1}{16}x^3 + \ldots$$

$$(1-2x)^{-\frac{1}{2}} = 1 + (-\tfrac{1}{2})(-2x) + \frac{-\frac{1}{2}\times-\frac{3}{2}}{1\times2}(-2x)^2 + \frac{-\frac{1}{2}\times-\frac{3}{2}\times-\frac{5}{2}}{1\times2\times3}(-2x)^3 + \ldots$$

$$= 1 + x + \tfrac{3}{2}x^2 + \tfrac{5}{2}x^3 + \ldots$$

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$$\sqrt{\frac{1-x}{1-2x}} = (1-\tfrac{1}{2}x-\tfrac{1}{8}x^2-\tfrac{1}{16}x^3+\ldots)(1+x+\tfrac{3}{2}x^2+\tfrac{5}{2}x^3+\ldots)$$
$$= 1+x+\tfrac{3}{2}x^2+\tfrac{5}{2}x^3-\tfrac{1}{2}x-\tfrac{1}{2}x^2-\tfrac{3}{4}x^3-\tfrac{1}{8}x^2-\tfrac{1}{8}x^3-\tfrac{1}{16}x^3+\ldots$$
$$= 1+\tfrac{1}{2}x+\tfrac{7}{8}x^2+\tfrac{25}{16}x^3+\ldots$$

First expansion is valid for $-1 < x < 1$

Second expansion is valid for $-1 < 2x < 1 \quad \Rightarrow -\tfrac{1}{2} < x < \tfrac{1}{2}$

so the whole expansion is valid for $-\tfrac{1}{2} < x < \tfrac{1}{2}$.

2. (i) $\dfrac{1}{\sqrt{1-2x}} = (1-2x)^{-\frac{1}{2}}$

$$= 1+(-\tfrac{1}{2})(-2x)+\frac{-\frac{1}{2}\times-\frac{3}{2}}{1\times 2}(-2x)^2+\ldots$$
$$= 1+x+\tfrac{3}{2}x^2+\ldots$$

(ii) $\dfrac{1}{\sqrt{0.8}} = \dfrac{1}{\sqrt{\frac{4}{5}}} = \sqrt{\dfrac{5}{4}} = \dfrac{\sqrt{5}}{\sqrt{4}} = \dfrac{\sqrt{5}}{2}$

(iii) Putting $x = 0.1$: $\dfrac{1}{\sqrt{1-2\times 0.1}} = 1+0.1+\tfrac{3}{2}(0.1)^2+\ldots$

$$\frac{1}{\sqrt{0.8}} = 1+0.1+0.015+\ldots$$
$$\frac{\sqrt{5}}{2} \approx 1.115$$
$$\sqrt{5} \approx 2.23$$

(iv) Percentage error $= \dfrac{\left|2.23-\sqrt{5}\right|}{\sqrt{5}}\times 100 = 0.271\%$