Section 1: The general binomial expansion

Solutions to Exercise level 1

1. (i) $(1-2x)^{-2} = 1 + (-2)(-2x) + \frac{-2\times-3}{1\times2}(-2x)^2 + \frac{-2\times-3\times-4}{1\times2\times3}(-2x)^3 + \ldots$

$= 1 + 4x + 12x^2 + 32x^3 + \ldots$

Valid for $-1 < 2x < 1$

$\Rightarrow -\frac{1}{2} < x < \frac{1}{2}$

(ii) $\sqrt{1+x} = (1+x)^{\frac{1}{2}}$

$= 1 + \frac{1}{2}x + \frac{\frac{1}{2}\times-\frac{1}{2}}{1\times2}x^2 + \frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{1\times2\times3}x^3 + \ldots$

$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \ldots$

Valid for $-1 < x < 1$

(iii) $\frac{1}{1+3x} = (1+3x)^{-1}$

$= 1 + (-1)(3x) + \frac{-1\times-2}{1\times2}(3x)^2 + \frac{-1\times-2\times-3}{1\times2\times3}(3x)^3 + \ldots$

$= 1 - 3x + 9x^2 - 27x^3 + \ldots$

Valid for $-1 < 3x < 1$

$\Rightarrow -\frac{1}{3} < x < \frac{1}{3}$

(iv) $(1-x)^{-\frac{2}{3}} = 1 + (-\frac{2}{3})(-x) + \frac{-\frac{2}{3}\times-\frac{5}{3}}{1\times2}(-x)^2 + \frac{-\frac{2}{3}\times-\frac{5}{3}\times-\frac{8}{3}}{1\times2\times3}(-x)^3 + \ldots$

$= 1 + \frac{2}{3}x + \frac{5}{9}x^2 + \frac{40}{81}x^3 + \ldots$

Valid for $-1 < x < 1$

2. (i) $\sqrt{4+x} = (4+x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1+\frac{x}{4}\right)^{\frac{1}{2}} = 2\left(1+\frac{x}{4}\right)^{\frac{1}{2}}$

$= 2\left(1 + \frac{1}{2}\left(\frac{x}{4}\right) + \frac{\frac{1}{2}\times-\frac{1}{2}}{1\times2}\left(\frac{x}{4}\right)^2 + \frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{1\times2\times3}\left(\frac{x}{4}\right)^3 + \ldots\right)$

$= 2\left(1 + \frac{x}{8} - \frac{1}{8}\left(\frac{x^2}{16}\right) + \frac{1}{16}\left(\frac{x^3}{64}\right) + \ldots\right)$

$= 2 + \frac{x}{4} - \frac{x^2}{64} + \frac{x^3}{512} + \ldots$

Valid for $-1 < \frac{x}{4} < 1$

$-4 < x < 4$

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(ii) $\dfrac{1}{2+x} = (2+x)^{-1} = 2^{-1}\left(1+\dfrac{x}{2}\right)^{-1} = \dfrac{1}{2}\left(1+\dfrac{x}{2}\right)^{-1}$

$$= \frac{1}{2}\left(1 - \frac{x}{2} + \frac{-1\times-2}{1\times2}\left(\frac{x}{2}\right)^2 + \frac{-1\times-2\times-3}{1\times2\times3}\left(\frac{x}{2}\right)^3 + \ldots\right)$$

$$= \frac{1}{2}\left(1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \ldots\right)$$

$$= \frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} - \frac{x^3}{16} + \ldots$$

Valid for $-1 < \dfrac{x}{2} < 1$

$$-2 < x < 2$$

(iii) $(3-2x)^{-2} = 3^{-2}\left(1-\dfrac{2x}{3}\right)^{-2} = \dfrac{1}{9}\left(1-\dfrac{2x}{3}\right)^{-2}$

$$= \frac{1}{9}\left(1 - 2\left(-\frac{2x}{3}\right) + \frac{-2\times-3}{1\times2}\left(-\frac{2x}{3}\right)^2 + \frac{-2\times-3\times-4}{1\times2\times3}\left(-\frac{2x}{3}\right)^3 + \ldots\right)$$

$$= \frac{1}{9}\left(1 + \frac{4x}{3} + 3\left(\frac{4x^2}{9}\right) - 4\left(-\frac{8x^3}{27}\right) + \ldots\right)$$

$$= \frac{1}{9} + \frac{4x}{27} + \frac{4x^2}{27} + \frac{32x^3}{243} + \ldots$$

Valid for $-1 < \dfrac{2x}{3} < 1$

$$-\frac{3}{2} < x < \frac{3}{2}$$

(iv) $(8-3x)^{\frac{1}{3}} = 8^{\frac{1}{3}}\left(1-\dfrac{3x}{8}\right)^{\frac{1}{3}} = 2\left(1-\dfrac{3x}{8}\right)^{\frac{1}{3}}$

$$= 2\left(1 + \frac{1}{3}\left(-\frac{3x}{8}\right) + \frac{\frac{1}{3}\times-\frac{2}{3}}{1\times2}\left(-\frac{3x}{8}\right)^2 + \frac{\frac{1}{3}\times-\frac{2}{3}\times-\frac{5}{3}}{1\times2\times3}\left(-\frac{3x}{8}\right)^3 + \ldots\right)$$

$$= 2\left(1 + \frac{1}{3}\left(-\frac{3x}{8}\right) - \frac{1}{9}\left(\frac{9x^2}{64}\right) + \frac{5}{81}\left(-\frac{27x^3}{512}\right) + \ldots\right)$$

$$= 2 - \frac{x}{4} - \frac{x^2}{32} - \frac{5x^3}{768} + \ldots$$

Valid for $-1 < \dfrac{3x}{8} < 1$

$$-\frac{8}{3} < x < \frac{8}{3}$$

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