

Section 3: Geometric sequences and series

Solutions to Exercise level 1

1. First term $a = 20$

$$\text{Common ratio } r = \frac{16}{20} = 0.8$$

$$10^{\text{th}} \text{ term} = ar^9 = 20 \times 0.8^9 = 2.68 \text{ (3 s.f.)}$$

2. First term $a = 1$

$$\text{Common ratio } r = 3$$

$$(i) \text{ 7th term} = ar^6 = 1 \times 3^6 = 729$$

$$(ii) S_8 = \frac{a(r^8 - 1)}{r - 1} = \frac{1(3^8 - 1)}{3 - 1} = 3280$$

3. First term $a = 2$

$$\text{Common ratio } r = 0.75$$

$$(i) \text{ 4th term} = ar^3 = 2 \times 0.75^3 = 0.84375$$

$$(ii) S_5 = \frac{a(1 - r^5)}{1 - r} = \frac{2(1 - 0.75^5)}{1 - 0.75} = 6.10 \text{ (3 s.f.)}$$

$$(iii) S_{\infty} = \frac{a}{1 - r} = \frac{2}{1 - 0.75} = 8$$

$$4. \text{ 3rd term} = 18 \quad \Rightarrow ar^2 = 18 \quad (1)$$

$$\text{6th term} = -60.75 \quad \Rightarrow ar^5 = -60.75 \quad (2)$$

$$\text{Dividing (2) by (1): } r^3 = \frac{-60.75}{18} = -3.375$$

$$r = -1.5$$

$$\text{Substituting into (1): } a(-1.5)^2 = 18$$

$$a = 8$$

The first term is 8 and the common ratio is -1.5.

Edexcel A level Maths Series 3 Exercise solutions

$$\begin{aligned}5. \text{ (i)} \quad S_{\infty} &= \frac{a}{1-r} \\ 5 &= \frac{2}{1-r} \\ 1-r &= 0.4 \\ r &= 0.6\end{aligned}$$

$$\text{(ii)} \quad S_{10} = \frac{2(1-r^{10})}{1-r} = \frac{2(1-0.6^{10})}{1-0.6} = 4.97 \text{ (3 s.f.)}$$

$$\text{(iii)} \quad \frac{2(1-0.6^n)}{1-0.6} > 4.99$$

$$1-0.6^n > 0.998$$

$$0.6^n < 0.002$$

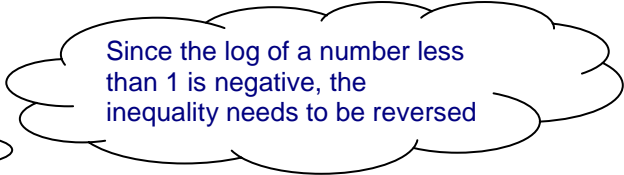
$$\log 0.6^n < \log 0.002$$

$$n \log 0.6 < \log 0.002$$

$$n > \frac{\log 0.002}{\log 0.6}$$

$$n > 12.2$$

13 terms are needed.



Since the log of a number less than 1 is negative, the inequality needs to be reversed

$$6. \text{ (i)} \quad u_5 = 12(1.5)^4 = 60.75$$

$$\begin{aligned}\text{(ii)} \quad S_{10} &= \frac{12(1.5^{10} - 1)}{1.5 - 1} \\ &= 1360 \text{ to nearest whole number.}\end{aligned}$$

$$7. \quad a = 28, r = \frac{1}{4}$$

$$\begin{aligned}\text{so } S_{\infty} &= \frac{28}{1 - \frac{1}{4}} \\ &= \frac{112}{3}\end{aligned}$$