Section 3: Geometric sequences and series

Solutions to Exercise level 1

1. First term $a = 20$

   Common ratio $r = \frac{16}{20} = 0.8$

   10^{th} term $= ar^9 = 20 \times 0.8^9 = 2.68$ (3 s.f.)

2. First term $a = 1$

   Common ratio $r = 3$

   (i) 7^{th} term $= ar^6 = 1 \times 3^6 = 729$

   (ii) $S_8 = \frac{a(r^8 - 1)}{r - 1} = \frac{1(3^8 - 1)}{3 - 1} = 3280$

3. First term $a = 2$

   Common ratio $r = 0.75$

   (i) 4^{th} term $= ar^3 = 2 \times 0.75^3 = 0.84375$

   (ii) $S_5 = \frac{a(1 - r^5)}{1 - r} = \frac{2(1 - 0.75^5)}{1 - 0.75} = 6.10$ (3 s.f.)

   (iii) $S_\infty = \frac{a}{1 - r} = \frac{2}{1 - 0.75} = 8$

4. 3^{rd} term $= 18$ $\Rightarrow ar^2 = 18$ (1)

   6^{th} term $= -60.75$ $\Rightarrow ar^5 = -60.75$ (2)

   Dividing (2) by (1): $r^3 = \frac{-60.75}{18} = -3.375$

   $r = -1.5$

   Substituting into (1): $a(-1.5)^2 = 18$

   $a = 8$

   The first term is 8 and the common ratio is -1.5.

Edexcel A level Maths Series 3 Exercise solutions

5. (i) $S_\infty = \dfrac{a}{1-r}$

$5 = \dfrac{2}{1-r}$

$1-r = 0.4$

$r = 0.6$

(ii) $S_{10} = \dfrac{2(1-r^{10})}{1-r} = \dfrac{2(1-0.6^{10})}{1-0.6} = 4.97$ (3 s.f.)

(iii) $\dfrac{2(1-0.6^n)}{1-0.6} > 4.99$

$1-0.6^n > 0.998$

$0.6^n < 0.002$

$\log 0.6^n < \log 0.002$

$n \log 0.6 < \log 0.002$

$n > \dfrac{\log 0.002}{\log 0.6}$

$n > 12.2$

13 terms are needed.

Since the log of a number less than 1 is negative, the inequality needs to be reversed

6. (i) $u_5 = 12(1.5)^4 = 60.75$

(ii) $s_{10} = \dfrac{12(1.5^{10}-1)}{1.5-1}$

$= 1360$ to nearest whole number.

7. $a = 28, r = \frac{1}{4}$

so $S_\infty = \dfrac{28}{1-\frac{1}{4}}$

$= \dfrac{112}{3}$