

Section 2: Arithmetic sequences and series

Solutions to Exercise level 3

1. (i) Scheme A is an arithmetic sequence, with $a = 20$ and $d = 4$.

$$\begin{aligned} \text{(ii)} \quad u_n &= 20 + (n-1)(4) \\ &= 4(4+n) \\ S_n &= \frac{n}{2}(2(20) + (n-1)(4)) \\ &= 2n(9+n) \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad S_n &= 2000 \Rightarrow 2n(9+n) = 2000 \\ &\Rightarrow n^2 + 9n - 1000 = 0 \\ &\Rightarrow n \approx 27.4, \text{ or } n \approx -36.4 \text{ (discard)} \\ &\text{and so Fred has fully paid by the end of the 28}^{\text{th}} \text{ month.} \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad S_{27} &= 54(36) = 1944 \\ &\text{so the final payment in month 28 should be } 2000 - 1944 = \text{£}56. \end{aligned}$$

2. (i) $a = 1000$, $d = 200$

$$\begin{aligned} S_{30} &= \frac{30}{2}(2000 + 29(200)) = 117000 \\ &\text{so Priya will save £117000} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad S_{30} &= \frac{30}{2}(2000 + 29d) = 150000 \\ &\Rightarrow (2000 + 29d) = 10000 \\ &\Rightarrow d = 275.86 \\ &\text{so she should make a yearly increase of £275.86} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad u_{10} &= 1000 + 9(275.86) = 3482.74 \\ S_{10} &= \frac{10}{2}(2000 + 9(275.86)) = 22413.70 \\ &\text{Remaining 20 years are an arithmetic series with} \\ &\quad a = 3482.74 + 350 = 3832.74 \\ &\quad d = 350 \\ S_{20} &= \frac{20}{2}(2(3832.74) + 19(350)) = 143154.80 \\ &\text{so total in 30 years} = 22413.70 + 143154.80 = \text{£}165568.50 \end{aligned}$$

3. (i) Total available = $13000 - 450 - 500 = 12050$
Expenditure is an arithmetic sequence, with $a = M_1$, $d = 100$

$$\begin{aligned} S_{12} &= \frac{12}{2}(2M_1 + (11)(100)) = 12050 \\ &\Rightarrow M_1 = 454.17 \end{aligned}$$

Edexcel A level Maths Sequences 2 Exercise solns

so Jane can plan to spend £454.17 in month 1.

(ii) After 5 months, she has spent

$$s_5 = \frac{5}{2}(2(454.17) + 4(100)) = 3270.85$$

$$\begin{aligned}\text{Her remaining money} &= 12050 - 3270.85 - 1000 \\ &= 7779.15\end{aligned}$$

Remaining 7 months are an arithmetic sequence, with $a = P$, $d = 75$

$$s_7 = \frac{7}{2}(2P + 6(75)) = 7779.15$$

$$\Rightarrow P = 886.31$$

so Jane can plan to spend £886.31 in month 6.