

Section 2: Arithmetic sequence and series

Solutions to Exercise level 2

1. (i) 7th term = $-2 \Rightarrow a + 6d = -2 \Rightarrow a = -2 - 6d$
 12th term = $8 \Rightarrow a + 11d = 8$
 Substituting first equation into second: $-2 - 6d + 11d = 8$
 $5d = 10$
 $d = 2$

$$a = -2 - 6d = -2 - 6 \times 2 = -14$$

The first term is -14 and the common difference is 2 .

(ii) $S_n = \frac{1}{2}n[2a + (n-1)d]$
 $364 = \frac{1}{2}n[2 \times -14 + 2(n-1)]$
 $364 = n(-14 + n - 1)$
 $364 = n^2 - 15n$
 $n^2 - 15n - 364 = 0$
 $(n - 28)(n + 13) = 0$
 Since n must be positive, $n = 28$.

2. (i) 1st term = $5 \times 1 - 3 = 2$
 2nd term = $5 \times 2 - 3 = 7$
 3rd term = $5 \times 3 - 3 = 12$
 Common difference = 5 .

(ii) $S_n = \frac{1}{2}n[2a + (n-1)d]$
 $S_{20} = \frac{1}{2} \times 20[2 \times 2 + 19 \times 5]$
 $= 10[4 + 95]$
 $= 10 \times 99$
 $= 990$

3. (i) 1st term = $S_1 = 4 + 2 = 6$

(ii) $S_2 = 4 \times 2 + 2 \times 2^2 = 8 + 8 = 16$
 2nd term = $16 - 6 = 10$
 Common difference = 4 .

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$$\begin{aligned} \text{(iii) } k^{\text{th}} \text{ term} &= a + (k-1)d \\ &= 6 + 4(k-1) \\ &= 6 + 4k - 4 \\ &= 4k + 2 \end{aligned}$$

$$\begin{aligned} 4. \text{ (i) } k^{\text{th}} \text{ term} &= 120 - 7(k-1) \\ 120 - 7(k-1) &< 0 \\ 120 - 7k + 7 &< 0 \\ 127 &< 7k \\ 7k &> 127 \\ k &> 18\frac{1}{7} \end{aligned}$$

The 19th term is the first term which is negative.

$$\begin{aligned} \text{(ii) } S_n &= \frac{1}{2}n[2a + (n-1)d] \\ &= \frac{1}{2}n[2 \times 120 - 7(n-1)] \\ &= \frac{1}{2}n[240 - 7n + 7] \\ \frac{1}{2}n(247 - 7n) &< 0 \\ \text{Since } n \text{ is positive, } 247 - 7n &< 0 \\ 247 &< 7n \\ 7n &> 247 \\ n &> 35\frac{2}{7} \end{aligned}$$

The sum of the first 36 terms is the first negative sum of terms.

$$\begin{aligned} 5. \text{ (i) } a &= 10, d = 2 \\ n^{\text{th}} \text{ term} &= 30 \\ 10 + 2(n-1) &= 30 \\ 2(n-1) &= 20 \\ n-1 &= 10 \end{aligned}$$

$$n = 11$$

He uses the bike for 30 minutes at the 11th session.

$$\begin{aligned} \text{(ii) } S_n &= \frac{1}{2}n[\text{first term} + \text{last term}] \\ &= \frac{1}{2} \times 11[10 + 30] \\ &= \frac{1}{2} \times 11 \times 40 \\ &= 11 \times 20 \\ &= 220 \end{aligned}$$

He has used the bike for a total of 220 minutes.

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$$6. \quad s_{20} = \frac{20}{2}(2a + 19d) = 1080 \\ \Rightarrow 2a + 19d = 108 \quad (1)$$

$$s_{30} = \frac{30}{2}(2a + 29d) = 2220 \\ \Rightarrow 2a + 29d = 148 \quad (2)$$

$$(2) - (1) \Rightarrow 10d = 40$$

$$\Rightarrow d = 4$$

$$(1) \Rightarrow a = 16$$

so the first terms are 16, 20, 24, 28, 32,

$$7. \quad \text{For A, } s_n = \frac{n}{2}(12 + (n-1)4)$$

$$= 4n + 2n^2$$

$$\text{For B, } s_n = \frac{n}{2}(50 + (n-1)2)$$

$$= 24n + n^2$$

When totals of series are the same:

$$4n + 2n^2 = 24n + n^2$$

$$n(n - 20) = 0$$

$$n = 0, \text{ or } n = 20$$

so the totals are equal after 20 terms, and series A total is greater after 21 terms.