

## Section 1: Definitions and notation

## Solutions to Exercise level 3

1. (i) Scheme A is an arithmetic sequence, with  $a = 20$  and  $d = 4$ .  
Scheme B is a geometric sequence, with  $a = 20$  and  $r = 1.12$

(ii)

month	scheme A		scheme B	
	payment	total	payment	total
1	20	20	20	20
2	24	44	22.40	42.40
3	28	72	25.09	67.49
4	32	104	28.10	95.59
5	36	140	31.47	127.06

(iii)

month	scheme A		scheme B	
	payment	total	payment	total
1	20	20	20	20
2	24	44	22.40	42.40
3	28	72	25.09	67.49
4	32	104	28.10	95.59
5	36	140	31.47	127.06
6	40	180	35.25	162.30
7	44	224	39.48	201.78
8	48	272	44.21	245.99
9	52	324	49.52	295.51
10	56	380	55.46	350.97
11	60	440	62.12	413.09
12	64	504	69.57	482.66
13	68	572	77.92	560.58
14	72	644	87.27	647.85
15	76	720	97.74	745.59
16	80	800	109.47	855.07
17	84	884	122.61	977.67
18	88	972	137.32	1114.99
19	92	1064		

so Scheme B pays off the debt first, in the 18<sup>th</sup> month, while Scheme A pays off the debt in the 19<sup>th</sup> month.

- (iv) The total paid by both schemes is nearly equal in month 14, when A has paid £644 and B has paid £647.85.

# Edexcel A level Maths Sequences 1 Exercise solutions

2. (i)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
new colonies	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610

Potentially, there are 610 new colonies in 2015.

(ii)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
new colonies	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610
total colonies	1	2	4	7	12	20	33	54	88	143	232	376	609	986	1596

(iii)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
new colonies	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610
total colonies	1	2	4	7	12	20	33	54	88	143	232	376	609	986	1596
ratio $y_{n+1}/y_n$	-----	1	2	1.5	1.667	1.6	1.625	1.615	1.619	1.618	1.618	1.618	1.618	1.618	1.618

The ratio converges (quickly) to a value near 1.618

(iv) The number of live colonies in 2015 is 1596.

$$\Rightarrow \text{No. in 2020} \approx 1596(1.618)^5 \approx 17698$$

By extending the table, the number in 2015 is expected to be 17710 (but don't forget that this would be limited by the predators!)

[Both the second and third rows are examples of a Fibonacci sequence, and whatever the initial two values of the sequence it is easy to show that the ratio converges to the larger solution of  $n^2 - n - 1 = 0$  which is

$\frac{1}{2}(1 + \sqrt{5})$ . So the ratios in both rows 2 and 3 converge to 1.618..... Test it!

This is also known as the 'golden ratio', and occurs in many places in nature. If you are unfamiliar with it, you may like to do a web search on it!]