Section 2: Composite and inverse functions

Solutions to Exercise level 2

1. (i) $f(x) \in \mathbb{R},\ f(x) \neq -2$

(ii) $y = \dfrac{3-2x}{x-5}$

$y(x-5) = 3-2x$

$xy - 5y = 3-2x$

$xy + 2x = 3+5y$

$x(y+2) = 3+5y$

$x = \dfrac{3+5y}{y+2}$

$f^{-1}(x) = \dfrac{3+5x}{x+2}$

Domain : $x \in \mathbb{R},\ x \neq -2$, range : $f(x) \in \mathbb{R}, f(x) \neq 5$

(iii) $\frac{1}{7}$

2. (i) $x \in \mathbb{R}, x \neq 0$

(ii) $f(x) = \dfrac{1}{3-x}$

$ff(x) = \dfrac{1}{3-\frac{1}{3-x}}$

$= \dfrac{3-x}{3(3-x)-1}$

$= \dfrac{3-x}{8-3x}$

Domain: $x \in \mathbb{R}, x \neq 2\frac{2}{3}$

(iii) $y = \dfrac{1}{3-x}$

$y(3-x)=1$

$3y-xy=1$

$3y-1=xy$

$x=\dfrac{3y-1}{y}$

$f^{-1}(x)=\dfrac{3x-1}{x}$

Domain: $x\in\mathbb{R}, x\neq 0$

Range: $f^{-1}(x)\in\mathbb{R}, f^{-1}(x)\neq 3$

3. (i) $y\geq -2$

(ii) $f(x)=x^2-2$

$gf(x)=\dfrac{1}{x^2-2}$,

Domain: $x\in\mathbb{R}, x\neq\pm\sqrt{2}$

(iii) $g(x)=\dfrac{1}{x}$

$gg(x)=\dfrac{1}{\frac{1}{x}}=x$

g(x) is its own inverse – a self-inverse function.

(iv) $f(x)=x^2-2$

$g(x)=\dfrac{1}{x}$

$fg(x)=\left(\dfrac{1}{x}\right)^2-2=\dfrac{1}{x^2}-2$

$y=\dfrac{1}{x^2}-2$

$y+2=\dfrac{1}{x^2}$

$\dfrac{1}{y+2}=x^2$

$x=\sqrt{\dfrac{1}{y+2}}$

$(fg)^{-1}(x)=\sqrt{\dfrac{1}{x+2}}$

Edexcel A level Maths Functions 2 Exercise solutions

4. (i) $e^{2x} = (e^x)^2$
$= gf(x)$

(ii) $e^{4x} = (e^{2x})^2$
$= ((e^x)^2)^2$
$= ggf(x)$

(iii) $e^{2x^2} = (e^{x^2})^2$
$= gfg(x)$

(iv) $e^{\sqrt{x}} = fg^{-1}(x)$

5. (i) $fg(x) = 2\sin x$

(ii) $fg(x) = 2\sin x$
$y = 2\sin x$
$\frac{y}{2} = \sin x$
$x = \arcsin(\frac{y}{2})$
$(fg)^{-1}(x) = \arcsin(\frac{x}{2})$