

## Section 2: Composite and inverse functions

## Solutions to Exercise level 1

1. (i)  $fg(x) = f(x^2) = 1 - x^2$

(ii)  $gh(x) = g\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 = \frac{1}{x^2}$

(iii)  $gfh(x) = gf\left(\frac{1}{x}\right) = g\left(1 - \frac{1}{x}\right) = \left(1 - \frac{1}{x}\right)^2$

(iv)  $fng(x) = fn(x^2) = f\left(\frac{1}{x^2}\right) = 1 - \frac{1}{x^2}$

(v)  $f^2(x) = ff(x) = f(1 - x) = 1 - (1 - x) = x$

(vi)  $h^2(x) = hh(x) = h\left(\frac{1}{x}\right) = x$

2. (i) The function  $\sqrt{x+2}$  means "add 2" followed by "square root", which is t followed by s. So this function is st(x).

(ii) The function  $\sqrt{x} + 2$  means "square root" followed by "add 2", which is s followed by t. So this function is ts(x).

(iii) The function  $x + 4$  can be expressed as "add 2" followed by "add 2". So this function is t<sup>2</sup>(x).

3. (i)  $y = 3x^3 - 1$

Interchanging x and y:  $x = 3y^3 - 1$

$$x + 1 = 3y^3$$

$$\frac{x+1}{3} = y^3$$

$$y = \sqrt[3]{\frac{x+1}{3}}$$

The inverse function is  $f^{-1}(x) = \sqrt[3]{\frac{x+1}{3}}$

(ii)  $y = \frac{x+2}{x-1}$

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Interchanging  $x$  and  $y$ :  $x = \frac{y+2}{y-1}$   
 $x(y-1) = y+2$   
 $xy - x = y+2$   
 $xy - y = 2+x$   
 $y(x-1) = x+2$   
 $y = \frac{x+2}{x-1}$   
The inverse function is  $f^{-1}(x) = \frac{x+2}{x-1}$

(iii)  $y = 1 - \frac{2}{x}$

Interchanging  $x$  and  $y$ :  $x = 1 - \frac{2}{y}$   
 $\frac{2}{y} = 1 - x$   
 $\frac{y}{2} = \frac{1}{1-x}$   
 $y = \frac{2}{1-x}$   
The inverse function is  $f^{-1}(x) = \frac{2}{1-x}$

4. (i)  $y = 2x + 1$

Interchanging  $x$  and  $y$ :  $x = 2y + 1$   
 $x - 1 = 2y$   
 $y = \frac{x-1}{2}$

$$f^{-1}(x) = \frac{x-1}{2}$$

(ii)  $y = \frac{2-x}{3}$

Interchanging  $x$  and  $y$ :  $x = \frac{2-y}{3}$   
 $3x = 2 - y$   
 $y = 2 - 3x$

$$g^{-1}(x) = 2 - 3x$$

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$$\begin{aligned} \text{(iii)} \quad fg(x) &= f\left(\frac{2-x}{3}\right) \\ &= 2\left(\frac{2-x}{3}\right) + 1 \\ &= \frac{4-2x+3}{3} \\ &= \frac{7-2x}{3} \end{aligned}$$

$$\text{(iv)} \quad y = \frac{7-2x}{3}$$

Interchanging  $x$  and  $y$ :

$$\begin{aligned} x &= \frac{7-2y}{3} \\ 3x &= 7-2y \\ 2y &= 7-3x \\ y &= \frac{7-3x}{2} \end{aligned}$$

$$(fg)^{-1} = \frac{7-3x}{2}$$

$$\begin{aligned} \text{(v)} \quad gf(x) &= g(2x+1) \\ &= \frac{2-(2x+1)}{3} \\ &= \frac{1-2x}{3} \end{aligned}$$

$$\text{(vi)} \quad y = \frac{1-2x}{3}$$

Interchanging  $x$  and  $y$ :

$$\begin{aligned} x &= \frac{1-2y}{3} \\ 3x &= 1-2y \\ 2y &= 1-3x \\ y &= \frac{1-3x}{2} \end{aligned}$$

$$(gf)^{-1} = \frac{1-3x}{2}$$

$$\begin{aligned} \text{(vii)} \quad f^{-1}g^{-1}(x) &= f^{-1}(2-3x) \\ &= \frac{(2-3x)-1}{2} \\ &= \frac{1-3x}{2} \end{aligned}$$

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$$\begin{aligned} \text{(viii) } g^{-1}f^{-1}(x) &= g^{-1}\left(\frac{x-1}{2}\right) \\ &= 2-3\left(\frac{x-1}{2}\right) \\ &= \frac{4-3x+3}{2} \\ &= \frac{7-3x}{2} \end{aligned}$$

$$(gf)^{-1}(x) = f^{-1}g^{-1}(x) \text{ and } (fg)^{-1}(x) = g^{-1}f^{-1}(x).$$