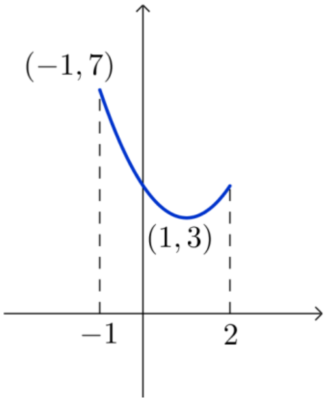
Section 1: Functions, graphs and transformations

Solutions to Exercise level 3 (Extension)

1. $f(x) = x^2 - 2x + 4 = (x-1)^2 + 3$ so there is a vertex at (1, 3).



So the minimum value of f(x) is 3.

The maximum is when $x = -1$

$f(-1) = (-1)^2 - 2\times -1 + 4 = 1 + 2 + 4 = 7$

The range is $3 \le f(x) \le 7$.

2. (i) $y = \dfrac{2x}{x^2+1}$

$y(x^2+1) = 2x$

$yx^2 - 2x + y = 0$

(ii) If there are real roots, $(2)^2 - 4(y)(y) \ge 0$

$4 - 4y^2 \ge 0$

$y^2 \le 1$

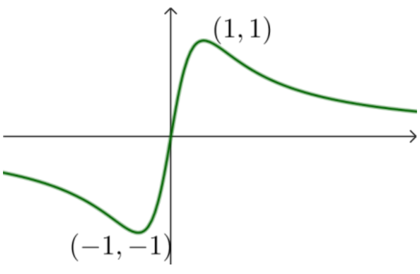
$-1 \le y \le 1$

(iii) Range is $-1 \le f(x) \le 1$

By inspection graph goes through (1, 1) and (-1, -1).

Also graph goes through origin

For large positive and negative x, $y \to 0$



Edexcel A level Maths Functions 1 Exercise solutions

3. $y = 2f(x-4)$

$8 = 2f(6-4)$

$4 = f(2)$

so $P = (2, 4)$

4. (i) $y = 2x^2 + 6 = 2(x^2 + 3)$ so T is a translation 3 units in the positive y direction, and S is a stretch scale factor 2 in the y-direction.

(ii) $y = \sqrt{4x+2} = \sqrt{4(x+\frac{1}{2})}$ so S is a stretch scale factor $\frac{1}{4}$ in the x-direction (taking $y = \sqrt{x}$ to $y = \sqrt{4x}$) and T is a translation of $-\frac{1}{2}$ units in the x-direction.

(iii) $y = \sqrt{4x} + 2 = 2\sqrt{x} + 2 = 2(\sqrt{x} + 1)$

so T is a translation of 1 unit in the positive y direction, and S is a stretch scale factor 2 in the y direction.

(iv) $y = 4x^2 - 4x + 1 = (2x-1)^2 = \left(2(x-\frac{1}{2})\right)^2$

so S is a stretch scale factor $\frac{1}{2}$ in the x-direction and T is a translation of $\frac{1}{2}$ unit in the positive x direction.