Section 1: Functions, graphs and transformations

Solutions to Exercise level 2

1. (i) (a) 6 has factors 1, 2, 3 and 6, so $d(6) = 4$.
(b) 5 has factors 1 and 5, so $d(5) = 2$.
(c) 16 has factors 1, 2, 4, 8 and 16, so $d(16) = 5$
(d) 13 has factors 1 and 13, so $d(13) = 2$.

(ii) Numbers which have only two factors (the number itself and 1) are prime numbers. So the set of numbers for which $d(n) = 2$ is the set of prime numbers.

(iii) Factors occur in pairs, so there are usually an even number of factors. However, in the case of a square number there is an odd number of factors, so the set of numbers for which $d(n)$ is odd is the set of square numbers.

2. (i) $x = 1$ must be excluded from the domain, since the function is not defined for this value.

(ii) (a) $f(2) = \frac{1}{2-1} = 1$

(b) $f(-3) = \frac{1}{-3-1} = -\frac{1}{4}$

(c) $f(0) = \frac{1}{0-1} = -1$

(iii) $f(x) = 2$

$$\frac{1}{x-1} = 2$$
$$1 = 2(x-1)$$
$$1 = 2x - 2$$
$$2x = 3$$
$$x = \tfrac{3}{2}$$

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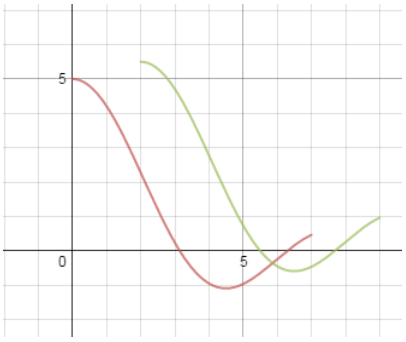
3. (i) $y = f(x-2)+0.5$

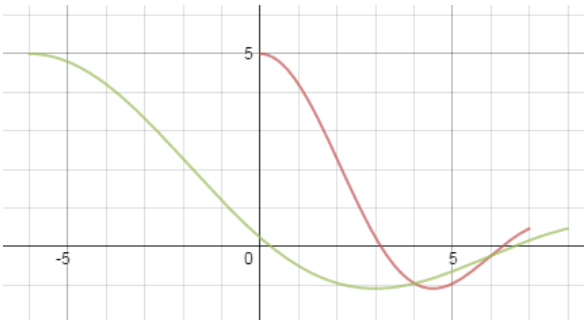


$f(x-2)$ translates the graph 2 in the positive x direction so adds 2 to all x-coordinates.

$+0.5$ translates the graph 0.5 units in the positive y direction so adds 0.5 to all y-coordinates.

Turning point (4.5 + 2, -1.1 + 0.5) so (6.5, -0.6),
Domain $2 \leq x \leq 9$,
Range $-0.6 \leq y \leq 5$.

(ii) $y = f(\frac{1}{2}x+3)$



$f(\frac{1}{2}x + 3)$ stretches by a factor of 2 in the x direction and translates the graph 3 units in the negative x direction so there is no change in the y-coordinates, and the x-coordinates have 3 subtracted from them and then they are multiplied by 2.

Turning point (2(4.5 – 3), -1.1) so (3, -1.1),
Domain $-6 \leq x \leq 8$,
Range $-1.1 \leq y \leq 5.0$

4. (i) Equation following translation is: $y = (x+2)^2 - 2(x+2)+1$
Equation following stretch is: $y = (3x+2)^2 - 2(3x+2)+1$
Equation following reflection is: $y = (-3x+2)^2 - 2(-3x+2)+1$
which simplifies to: $y = 9x^2 - 6x + 1$

(ii) Original has line of symmetry $x = 1$, then $x = -1$, then $x = -\frac{1}{3}$, then $x = \frac{1}{3}$.

(iii) $\left(\frac{1}{3}, 0\right)$

5. (i) The graph of $y = 1 + \sin(x + 30°)$ is obtained from the graph of $y = \sin x$ by a translation of $\begin{pmatrix} -30° \\ 1 \end{pmatrix}$.

(ii) The graph of $y = -3\sin\frac{1}{2}x$ is obtained from the graph of $y = \sin x$ by a stretch of scale factor 2 parallel to the x-axis, a stretch of scale factor 3 parallel to the y-axis, and a reflection in the x-axis.
(These may be carried out in any order).

6. (i) $y = 5x^2 - 15x + 4$

$= 5(x^2 - 3x) + 4$

$= 5\left[(x - \frac{3}{2})^2 - \frac{9}{4}\right] + 4$

$= 5(x - \frac{3}{2})^2 - \frac{29}{4}$

(ii) translation of $\begin{pmatrix} -\frac{3}{2} \\ \frac{29}{4} \end{pmatrix}$ then a stretch of scale factor 0.2 parallel to the y-axis.

(iii) Translation of $\begin{pmatrix} -\frac{3}{2} \\ \frac{29}{4} \end{pmatrix}$ gives

$y = 5(x + \frac{3}{2})^2 - 15(x + \frac{3}{2}) + 4 + \frac{29}{4}$

$= 5(x + \frac{3}{2})^2 - 15(x + \frac{3}{2}) + \frac{45}{4}$

Stretch of scale factor 0.2 parallel to the y-axis gives

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$$y = \frac{1}{5}(5(x+\tfrac{3}{2})^2 - 15(x+\tfrac{3}{2}) + \frac{45}{4})$$

$$= (x+\tfrac{3}{2})^2 - 3(x+\tfrac{3}{2}) + \frac{9}{4}$$

$$= x^2 + 3x + \frac{9}{4} - 3x - \frac{9}{2} + \frac{9}{4}$$

$$= x^2$$