Section 1: Functions, graphs and transformations

Solutions to Exercise level 1

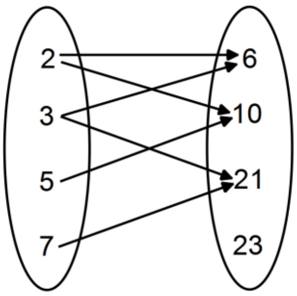
1. (a) (i) The mapping is one-to-many.
(ii) The mapping is not a function.

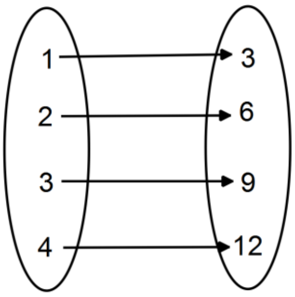
(b) (i) The mapping is one-to-one.
(ii) The mapping is a function.

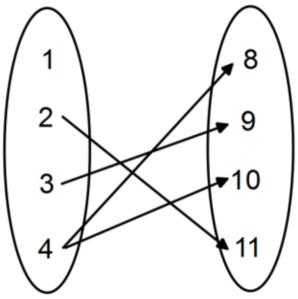
(c) (i) The mapping is many-to-many.
(ii) The mapping is not a function.

(d) (i) The mapping is many-to-one.
(ii) The mapping is a function.

2. (i) (a) (b) (c)

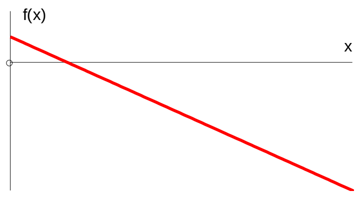






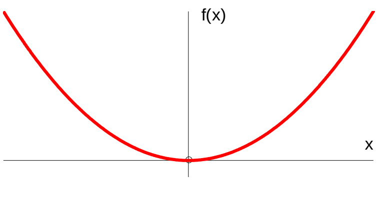
(ii) (a) many-to-many (b) one-to-one (c) one-to-many
(iii) (a) no (b) yes (c) no

3. (i) $f(x) = 1 - 3x$ where $x > 0$



The range is $f(x) < 1$.

(ii) $f(x) = x^2$ where $x \in \mathbb{R}$



The range is $f(x) \geq 0$.

(iii) $f(x) = \dfrac{1}{1+x^2}$ where $-1 \le x \le 1$

The largest possible value of f(x) is when x = 0, where f(x) = 1.

The smallest possible value of f(x) is when $x = \pm 1$, where $f(x) = \frac{1}{2}$.

The range is $\frac{1}{2} \le f(x) \le 1$.

4. (i) $f(x) \in \mathbb{Q}^+, f(x) \ge 3$

(ii) $f(x) \in \mathbb{R}, -9 \le f(x) < 21$

(iii) $f(x) \in \mathbb{R}, -1 \le f(x) < 1$

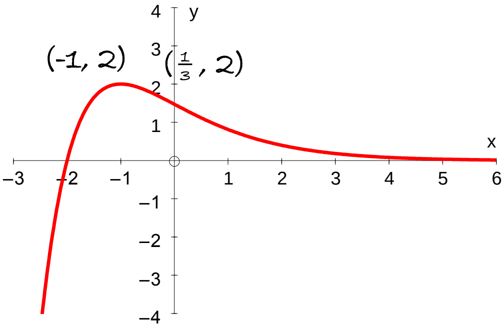
(iv) $f(x) \in \mathbb{R}, f(x) > 0$

(v) $f(x) \in \mathbb{R}$

(vi) $f(x) \in \mathbb{R}, f(x) \ge 0$

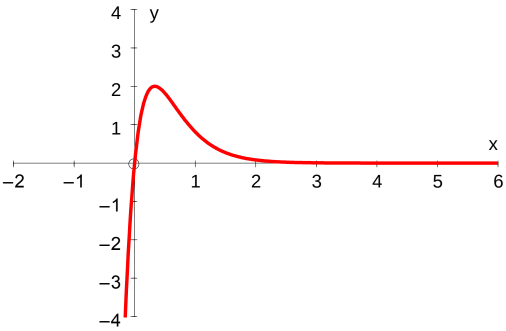
5. (i) $y = f(x+2)$

This graph is obtained from the graph of $y = f(x)$ by a translation of 2 units to the left. The turning point is (-1, 2).



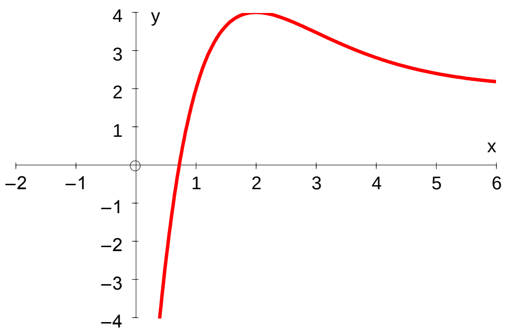
(ii) $y = f(3x)$

This graph is obtained (2, 4) from the graph of $y = f(x)$ by a stretch, scale factor $\frac{1}{3}$ parallel to the x-axis. The turning point is ($\frac{1}{3}$, 2).



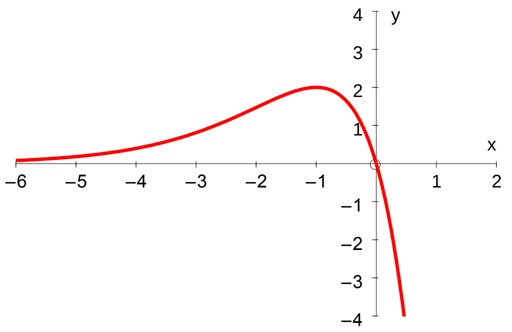
(iii) $y = f(x-1) + 2$

This graph is obtained from the graph of $y = f(x)$ by a translation through $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. The turning point is (2, 4).



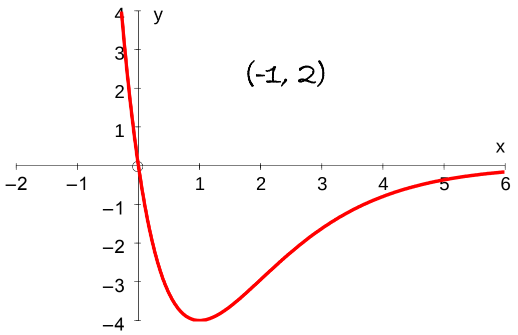
(iv) $y = f(-x)$

This graph is obtained from the graph of $y = f(x)$ by a reflection in the y-axis. The turning point is (-1, 2).



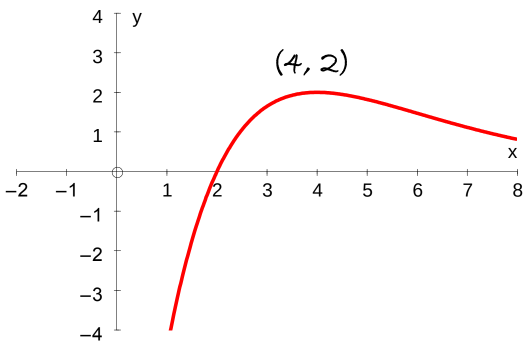
(v) $y = -2f(x)$

This graph is obtained from the graph of $y = f(x)$ by a reflection in the x-axis and a stretch scale factor 2 parallel to the y-axis. The turning point is (1, -4).



(vi) $y = f(\frac{1}{2}x - 1)$

This graph is obtained from the graph of $y = f(x)$ by a translation of 1 unit to the right followed by a stretch, scale factor 2, parallel to the x-axis. The turning point is (4, 2).



6\. (i) A translation through $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ maps the graph of y = f(x) to the graph of y = f(x – 3) – 1.

So the graph of $y = x^2$ is mapped to the graph of

$$\begin{aligned} y &= (x-3)^2 - 1 \\ &= x^2 - 6x + 9 - 1 \\ &= x^2 - 6x + 8 \end{aligned}$$

(ii) A stretch parallel to the x-axis, scale factor $\frac{1}{2}$ maps the graph of y = f(x) to the graph of y = f(2x).

So the graph of $y = x^2$ is mapped to the graph of

$$y = (2x)^2 = 4x^2$$

(iii) A reflection in the y-axis maps the graph of y = f(x) to the graph of y = f(-x).

So the graph of $y = x^2$ is mapped to the graph of

$$y = (-x)^2 = x^2$$

(iv) A stretch parallel to the y-axis, scale factor 3 maps the graph of y = f(x) to the graph of y = 3f(x).

So the graph of $y = x^2$ is mapped to the graph of $y = 3x^2$

(v) A translation through $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ maps the graph of y = f(x) to the graph of y = f(x + 2). So the graph of $y = x^2$ is mapped to the graph of

$$y = (x+2)^2$$

A reflection in the x-axis maps the graph of y = f(x) to the graph of y = -f(x). So the graph of $y = (x+2)^2$ is mapped to the graph of

$$\begin{aligned} y &= -(x+2)^2 \\ &= -(x^2 + 4x + 4) \\ &= -x^2 - 4x - 4 \end{aligned}$$

(vi) A stretch parallel to the y-axis, scale factor 2 maps the graph of $y = f(x)$ to the graph of $y = 2f(x)$. So the graph of $y = x^2$ is mapped to the graph of $y = 2x^2$.

A translation through $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ maps the graph of $y = f(x)$ to the graph of $y = f(x - 1) + 2$. So the graph of $y = 2x^2$ is mapped to the graph of $y = 2(x-1)^2 + 2$

A reflection in the y-axis maps the graph of $y = f(x)$ to the graph of $y = f(-x)$. So the graph of $y = 2(x-1)^2 + 2$ is mapped to the graph of

$$\begin{aligned} y &= 2(-x-1)^2 + 2 \\ &= 2(x^2 + 2x + 1) + 2 \\ &= 2x^2 + 4x + 2 + 2 \\ &= 2x^2 + 4x + 4 \end{aligned}$$