Section 2: Rational expressions

Solutions to Exercise level 2

1. (i) $\dfrac{2}{x+3}+\dfrac{1}{x-5}=\dfrac{2(x-5)}{(x+3)(x-5)}+\dfrac{x+3}{(x+3)(x-5)}$

$=\dfrac{2x-10+x+3}{(x+3)(x-5)}$

$=\dfrac{3x-7}{(x+3)(x-5)}$

(ii) $\dfrac{5}{2x-3}-\dfrac{2}{4x+1}=\dfrac{5(4x+1)}{(2x-3)(4x+1)}-\dfrac{2(2x-3)}{(2x-3)(4x+1)}$

$=\dfrac{20x+5-4x+6}{(2x-3)(4x+1)}$

$=\dfrac{16x+11}{(2x-3)(4x+1)}$

(iii) $t-\dfrac{5}{3t-1}=\dfrac{t(3t-1)}{3t-1}-\dfrac{5}{3t-1}$

$=\dfrac{3t^2-t-5}{3t-1}$

2. (i) $\dfrac{1}{2(x+5)}-\dfrac{1}{4(x+1)}=\dfrac{2(x+1)}{4(x+5)(x+1)}-\dfrac{x+5}{4(x+5)(x+1)}$

$=\dfrac{2x+2-x-5}{4(x+5)(x+1)}$

$=\dfrac{x-3}{4(x+5)(x+1)}$

(ii) $\dfrac{3u}{u^2-4u+4}+\dfrac{1}{2-u}=\dfrac{3u}{(u-2)^2}-\dfrac{1}{u-2}$

$=\dfrac{3u}{(u-2)^2}-\dfrac{u-2}{(u-2)^2}$

$=\dfrac{3u-u+2}{(u-2)^2}$

$=\dfrac{2u+2}{(u-2)^2}$

Edexcel A level Maths Algebra 2 Exercise solutions

3. (i) $\frac{3x}{4} - \frac{x}{6} = 7$

$\frac{9x}{12} - \frac{2x}{12} = 7$

$\frac{7x}{12} = 7$

$x = 12$

(ii) $\frac{30}{2x-5} + \frac{27}{2x+1} = 13$

$30(2x+1) + 27(2x-5) = 13(2x-5)(2x+1)$

$60x + 30 + 54x - 135 = 13(4x^2 - 8x - 5)$

$114x - 105 = 52x^2 - 104x - 65$

$52x^2 - 218x + 40 = 0$

$26x^2 - 109x + 20 = 0$

$(x-4)(26x-5) = 0$

$x = 4$ or $x = \frac{5}{26}$

4. (i) Since a cubic is being divided by a linear expression, the quotient is quadratic and the remainder constant.

$\frac{3x^3 - x^2 + 2x - 4}{x+2} = Ax^2 + Bx + C + \frac{D}{x+2}$

$3x^3 - x^2 + 2x - 4 = (Ax^2 + Bx + C)(x+2) + D$

$= Ax^3 + (2A+B)x^2 + (2B+C)x + 2C + D$

Equating coefficients of x^3: $A = 3$

Equating coefficients of x^2: $2A + B = -1 \Rightarrow B = -7$

Equating coefficients of x: $2B + C = 2 \Rightarrow C = 16$

Equating constant terms: $2C + D = -4 \Rightarrow D = -36$

$\frac{3x^3 - x^2 + 2x - 4}{x+2} = 3x^2 - 7x + 16 - \frac{36}{x+2}$

(ii) Since a cubic is being divided by a quadratic, the quotient is linear and the remainder linear.

$\frac{x^3}{x^2+2} = Ax + B + \frac{Cx + D}{x^2+2}$

$x^3 = (Ax + B)(x^2 + 2) + Cx + D$

$= Ax^3 + Bx^2 + (2A + C)x + 2B + D$

Equating coefficients of x^3: $A = 1$

Equating coefficients of x^2: $B = 0$

Equating coefficients of x: $2A + C = 0 \Rightarrow C = -2$

Equating constant terms: $2B + D = 0 \Rightarrow D = 0$

$$\frac{x^3}{x^2+2} = x - \frac{2x}{x^2+2}$$

(iii) Since a linear expression is being divided by a linear expression, the quotient is a constant and the remainder a constant.

$$\frac{6x-2}{2x+3} = A + \frac{B}{2x+3}$$

$$6x-2 = A(2x+3)+B$$

$$= 2Ax + 3A + B$$

Comparing coefficients of x: $2A = 6 \Rightarrow A = 3$

Comparing constant terms: $3A + B = -2 \Rightarrow B = -11$

$$\frac{6x-2}{2x+3} = 3 - \frac{11}{2x+3}$$