Section 3: Partial fractions

Solutions to Exercise level 2

1. (i) $\dfrac{x+7}{(x+1)(x-2)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x-2}$

$x+7 \equiv A(x-2)+B(x+1)$

Putting $x=-1 \quad \Rightarrow 6=-3A \quad \Rightarrow A=-2$

Putting $x=2 \quad \Rightarrow 9=3B \quad \Rightarrow B=3$

$\dfrac{x+7}{(x+1)(x-2)} \equiv \dfrac{3}{x-2} - \dfrac{2}{x+1}$

(ii) $\dfrac{x+1}{x^2-4} \equiv \dfrac{x+1}{(x-2)(x+2)} \equiv \dfrac{A}{x-2} + \dfrac{B}{x+2}$

$x+1 \equiv A(x+2)+B(x-2)$

Putting $x=2 \quad \Rightarrow 3=4A \quad \Rightarrow A=\frac{3}{4}$

Putting $x=-2 \quad \Rightarrow -1=-4B \quad \Rightarrow B=\frac{1}{4}$

$\dfrac{x+1}{x^2-4} \equiv \dfrac{3}{4(x-2)} + \dfrac{1}{4(x+2)}$

2. (i) $\dfrac{4}{(1+3x)(1+x)^2} \equiv \dfrac{A}{1+3x} + \dfrac{B}{1+x} + \dfrac{C}{(1+x)^2}$

$4 \equiv A(1+x)^2 + B(1+3x)(1+x) + C(1+3x)$

Putting $x=-\frac{1}{3} \quad \Rightarrow 4=\frac{4}{9}A \quad \Rightarrow A=9$

Putting $x=-1 \quad \Rightarrow 4=-2C \quad \Rightarrow C=-2$

Equating coefficients of $x^2 \Rightarrow 0=A+3B$

$\Rightarrow 3B=-9$

$\Rightarrow B=-3$

$\dfrac{4}{(1+3x)(1+x)^2} \equiv \dfrac{9}{1+3x} - \dfrac{3}{1+x} - \dfrac{2}{(1+x)^2}$

(ii) $\dfrac{13-7x}{(x-2)^2(x-3)} \equiv \dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{x-3}$

$13-7x \equiv A(x-2)(x-3)+B(x-3)+C(x-2)^2$

Putting $x=2 \quad \Rightarrow -1=-B \quad \Rightarrow B=1$

Putting $x=3 \quad \Rightarrow -8=C$

Equating coefficients of $x^2 \Rightarrow 0=A+C \quad \Rightarrow A=8$

$\dfrac{13-7x}{(x-2)^2(x-3)} \equiv \dfrac{8}{x-2} + \dfrac{1}{(x-2)^2} - \dfrac{8}{x-3}$

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3. (i) $\dfrac{x+5}{(x-1)(x+2)} \equiv \dfrac{A}{x-1} + \dfrac{B}{x+2}$

$x+5 \equiv A(x+2) + B(x-1)$

Putting $x=1 \quad \Rightarrow 6 = 3A \quad \Rightarrow A = 2$

Putting $x=-2 \quad \Rightarrow 3 = -3B \quad \Rightarrow B = -1$

$$\frac{x+5}{(x-1)(x+2)} \equiv \frac{2}{x-1} - \frac{1}{x+2}$$
$$= 2(x-1)^{-1} - (2+x)^{-1}$$
$$= -2(1-x)^{-1} - 2^{-1}(1+\tfrac{1}{2}x)^{-1}$$
$$= -2(1-x)^{-1} - \tfrac{1}{2}(1+\tfrac{1}{2}x)^{-1}$$

$$(1-x)^{-1} = 1 + (-1)(-x) + \frac{-1\times-2}{1\times2}(-x)^2 + \frac{-1\times-2\times-3}{1\times2\times3}(-x)^3 + \ldots$$
$$= 1 + x + x^2 + x^3 + \ldots$$
$$(1+\tfrac{1}{2}x)^{-1} = 1 + (-1)(\tfrac{1}{2}x) + \frac{-1\times-2}{1\times2}(\tfrac{1}{2}x)^2 + \frac{-1\times-2\times-3}{1\times2\times3}(\tfrac{1}{2}x)^3 + \ldots$$
$$= 1 - \tfrac{1}{2}x + \tfrac{1}{4}x^2 - \tfrac{1}{8}x^3 + \ldots$$
$$\frac{x+5}{(x-1)(x+2)} = -2(1 + x + x^2 + x^3 + \ldots) - \tfrac{1}{2}(1 - \tfrac{1}{2}x + \tfrac{1}{4}x^2 - \tfrac{1}{8}x^3 + \ldots)$$
$$= -2 - 2x - 2x^2 - 2x^3 - \tfrac{1}{2} + \tfrac{1}{4}x - \tfrac{1}{8}x^2 + \tfrac{1}{16}x^3 + \ldots$$
$$= -\tfrac{5}{2} - \tfrac{7}{4}x - \tfrac{17}{8}x^2 - \tfrac{31}{16}x^3 + \ldots$$

First expansion is valid for $-1 < x < 1$

Second expansion is valid for $-1 < \tfrac{1}{2}x < 1 \quad \Rightarrow -2 < x < 2$

so the whole expansion is valid for $-1 < x < 1$.

(ii) $\dfrac{7}{(1-2x)(x+3)} \equiv \dfrac{A}{1-2x} + \dfrac{B}{x+3}$

$7 = A(x+3) + B(1-2x)$

Putting $x=-3 \quad \Rightarrow 7 = 7B \quad \Rightarrow B = 1$

Putting $x=\tfrac{1}{2} \quad \Rightarrow 7 = \tfrac{7}{2}A \quad \Rightarrow A = 2$

$$\frac{7}{(1-2x)(x+3)} \equiv \frac{2}{1-2x} + \frac{1}{x+3}$$
$$= 2(1-2x)^{-1} + (3+x)^{-1}$$
$$= 2(1-2x)^{-1} + 3^{-1}(1+\tfrac{1}{3}x)^{-1}$$
$$= 2(1-2x)^{-1} + \tfrac{1}{3}(1+\tfrac{1}{3}x)^{-1}$$

$$(1-2x)^{-1} = 1 + (-1)(-2x) + \frac{-1\times-2}{1\times2}(-2x)^2 + \frac{-1\times-2\times-3}{1\times2\times3}(-2x)^3 + \ldots$$
$$= 1 + 2x + 4x^2 + 8x^3 + \ldots$$

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$$(1+\tfrac{1}{2}x)^{-1} = 1+(-1)(\tfrac{1}{3}x)+\frac{-1\times-2}{1\times2}(\tfrac{1}{3}x)^2+\frac{-1\times-2\times-3}{1\times2\times3}(\tfrac{1}{3}x)^3+\ldots$$

$$= 1-\tfrac{1}{3}x+\tfrac{1}{9}x^2-\tfrac{1}{27}x^3+\ldots$$

$$\frac{7}{(1-2x)(x+3)} = 2(1+2x+4x^2+8x^3+\ldots)$$

$$+\tfrac{1}{3}(1-\tfrac{1}{3}x+\tfrac{1}{9}x^2-\tfrac{1}{27}x^3+\ldots)$$

$$= 2+4x+8x^2+16x^3+\tfrac{1}{3}-\tfrac{1}{9}x+\tfrac{1}{27}x^2-\tfrac{1}{81}x^3+\ldots$$

$$= \tfrac{7}{3}+\tfrac{35}{9}x+\tfrac{217}{27}x^2+\tfrac{1295}{81}x^3+\ldots$$

First expansion is valid for $-1<2x<1 \quad \Rightarrow -\tfrac{1}{2}<x<\tfrac{1}{2}$

Second expansion is valid for $-1<\tfrac{1}{3}x<1 \quad \Rightarrow -3<x<3$

so the whole expansion is valid for $-\tfrac{1}{2}<x<\tfrac{1}{2}$.