Section 2: General equations

Section test

(Throughout this test, unless instructed otherwise, take g = 9.8 ms^{-2} and round answers, where necessary, to 3 s.f.)

1. **In this question, take g = 10 ms^{-2}.**
   A particle P is projected from a point A at the top of a cliff, 52 m vertically above sea level. It moves freely under gravity until it strikes the sea at a point S. The initial velocity components are 24 ms^{-1} horizontally and 7 ms^{-1} vertically.

   Find the time of flight of the particle.
   Find the horizontal distance AS.
   Find the speed of the particle when it hits the sea.

2. A bullet is fired with an initial velocity of 600 ms^{-1} in a direction making an angle of 25° with the horizontal. Find its range over horizontal ground.

3. Find the greatest possible range of a projectile which is fired at 60 ms^{-1} inside a tunnel which is 4.8m high with a level floor.

4. Over level ground, the greatest range of a gun is 25 km. Find the muzzle velocity of a bullet leaving the barrel of this gun.

5. A particle is projected such that its range over level ground is three times the maximum height of its path. Find the angle of projection.

6. A particle is projected from a point O and passes through a point P when travelling horizontally. P is 10 m horizontally and 8 m vertically from O.

   Find the angle of projection, to the nearest degree.
   Find the magnitude of the initial velocity.

7. A gun fires a shell at 210 ms^{-1}. What is the lowest angle of elevation at which a shell should be fired to hit a target 3.6 km away, at the same level as the gun.

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Solutions to section test

1) Taking the origin as point A, the time of flight is the time at which $y = -52$.

Vertically: $u = 7$, $g = -10$, $s = -52$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$-52 = 7t - \frac{1}{2} \times 10t^2$

$5t^2 - 7t - 52 = 0$

$(5t + 13)(t - 4) = 0$

$t = -\frac{13}{5}$ or $t = 4$

Time of flight must be positive, so time of flight of the particle is 4 seconds.

The horizontal distance AS is the horizontal distance travelled in 4 seconds.
The horizontal velocity $= 24$ ms^{-1}
so the horizontal distance $x = 24 \times 4 = 96$ m.

The horizontal velocity when it hits the sea $= 24$ ms^{-1}.
The vertical velocity when it hits the sea is given by

$$v_y^2 = u_y^2 - 2gs$$
$$= 7^2 + 2 \times -10 \times -52$$
$$= 1089$$
$$v_y = 33$$

Speed of particle $= \sqrt{24^2 + 33^2} = 40.8$ ms^{-1} (3 s.f.).

2) Vertically: $y = vt \sin\alpha - \frac{1}{2}gt^2$

$0 = 600t \sin 25^\circ - \frac{1}{2} \times 9.8t^2$

$0 = t(600 \sin 25^\circ - 4.9t)$

$t = 0$ or $t = \dfrac{600 \sin 25^\circ}{4.9}$

Horizontally: $x = vt \cos\alpha$

$$= 600 \times \frac{600 \sin 25^\circ}{4.9} \times \cos 25^\circ$$
$$= 28100 \text{ m (3 s.f.)}$$

The range of the bullet is 28.1 km.

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3) Greatest possible range will occur if the greatest height is 4.8 m.

At greatest height: $v_y^2 = u_y^2 - 2gs$

$0 = (60\sin\theta)^2 - 2 \times 9.8 \times 4.8$

$3600\sin^2\theta = 94.08$

$\sin^2\theta = \frac{94.08}{3600}$

$\theta = 9.303°$

For time of flight: $y = ut\sin\theta - \frac{1}{2}gt^2$

$0 = t(60\sin\theta - 4.9t)$

$t = \frac{60\sin\theta}{4.9}$

Range $= 60t\cos\theta$

$= 60 \times \frac{60\sin\theta}{4.9} \times \cos\theta$

$= 117$ m (3 s.f.)

4) The greatest range is the range when the angle of projection = 45°.

Vertically: $y = Vt\sin 45° - \frac{1}{2}gt^2$

$0 = t\left(\frac{V}{\sqrt{2}} - 4.9t\right)$

$t = \frac{V}{4.9\sqrt{2}}$

Horizontally: $25000 = Vt\cos 45°$

$25000 = V \times \frac{V}{4.9\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$V^2 = 25000 \times 9.8$

$V = 495$

The muzzle velocity is 495 ms^{-1} (3 s.f.)

5) Range $= \frac{u^2\sin 2\theta}{g}$

Maximum height $= \frac{u^2\sin^2\theta}{2g}$

Range = 3 × maximum height

$$\frac{u^2 \sin 2\theta}{g} = 3 \times \frac{u^2 \sin^2 \theta}{2g}$$

$2 \sin 2\theta = 3 \sin^2 \theta$

$4 \sin \theta \cos \theta - \sin^2 \theta = 0$

$\sin \theta (4 \cos \theta - 3 \sin \theta) = 0$

$\sin \theta = 0$ or $4 \cos \theta = 3 \sin \theta$

$\theta = 0$ or $\tan \theta = \frac{4}{3}$

$\theta = 53.1°$

6) When travelling horizontally, the particle is at its greatest height.

Maximum height $= \frac{u^2 \sin^2 \theta}{2g} \Rightarrow \frac{u^2 \sin^2 \theta}{2g} = 8$ (1)

When at its greatest height, horizontal distance travelled is 10 m, so horizontal range = 20 m.

Range $= \frac{u^2 \sin 2\theta}{g} \Rightarrow \frac{u^2 \sin 2\theta}{g} = 20$ (2)

(1) ÷ (2):

$$\frac{\sin^2 \theta}{2 \sin 2\theta} = \frac{8}{20}$$

$$\frac{\sin^2 \theta}{4 \sin \theta \cos \theta} = \frac{2}{5}$$

$$\tan \theta = \frac{8}{5}$$

$\theta = 58°$ (2 s.f.)

From above, $\frac{u^2 \sin^2 \theta}{2g} = 8$

$$u^2 = \frac{16g}{\sin^2 \theta} = \frac{16 \times 9.8}{\sin^2 57.995}$$

$u = 14.8$ ms^{-1} (3 s.f.)

7) Range $= \frac{u^2 \sin 2\theta}{g}$

$3600 = \frac{210^2 \sin 2\theta}{9.8}$

$\sin 2\theta = 0.8$

$2\theta = 53.13°$ or $126.87°$

$\theta = 26.6°$ or $63.4°$

The lowest angle is 26.6°.