Section 1: Introduction

Notes and Examples

These notes contain subsections on

- Modelling assumptions
- General strategy for projectile questions
- Components of the velocity
- Finding the time of flight, range and maximum height

Modelling assumptions

When you are working with projectiles, you will usually make the following assumptions:

- The projectile is a particle
- It is not powered (so the only force acting is gravity
- The air has no effect on its motion (no air resistance)

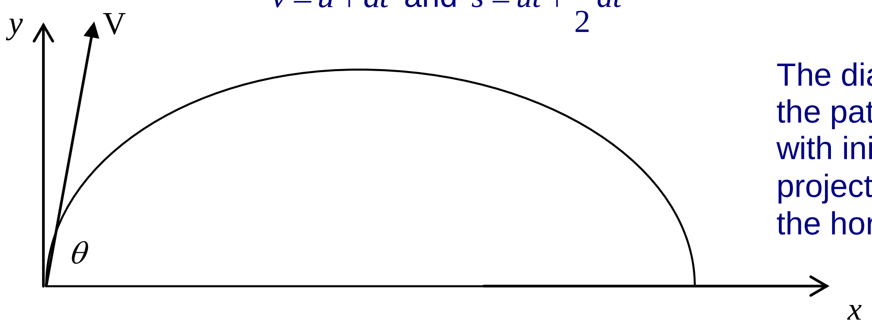
Without these assumptions, analysing projectile motion would be much harder. In many situations these assumptions will not make a significant difference to the final answer, so they are reasonable. However, throwing a flat sheet of paper, for example, could not usefully be analysed without taking account of the effects of air resistance.

General strategy for projectile questions

Splitting the velocity into two components, horizontal and vertical, is the standard way to solve projectiles questions.

The equations of motion are then applied to each component of velocity. The main ones used are:-

$v = u + at$ and $s = ut + \frac{1}{2}at^2$



The diagram shows the path of a projectile with initial velocity V, projected at angle θ to the horizontal.

Vertically

$a_y = -g$

$u_y = V\sin\theta$

vertical velocity (using $v = u + at$)

Horizontally

$a_x = 0$

$u_x = V\cos\theta$

horizontal velocity (using $v = u + at$)

$$v_y = V\sin\theta - gt \qquad v_x = V\cos\theta$$

$$y = Vt\sin\theta - \tfrac{1}{2}gt^2 \qquad x = Vt\cos\theta$$

vertical displacement (using $s = ut + \frac{1}{2}at^2$)

horizontal displacement (using $s = ut + \frac{1}{2}at^2$)

where t is the time of flight.

Sometimes it may be more efficient to work in vector form, so that you are dealing with the vertical and horizontal motion at the same time.

Applying the vector equation $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ gives

$$\mathbf{v} = \begin{pmatrix} V\cos\theta \\ V\sin\theta \end{pmatrix} + \begin{pmatrix} 0 \\ -g \end{pmatrix} t$$

Applying the vector equation $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ gives

$$\mathbf{s} = \begin{pmatrix} V\cos\theta \\ V\sin\theta \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -g \end{pmatrix} t^2$$

Using column vectors rather than the $a\mathbf{i} + b\mathbf{j}$ form of a vector is often clearer since you can then read each separate equation off.

Components of the velocity

The vertical component

When the vertical component of velocity is positive, the particle is rising.

- At the instant the vertical component of velocity is 0, the particle is at maximum height.
- When the vertical component of velocity is negative, the particle is falling.

The horizontal component

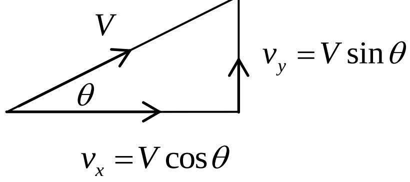
Remember: THE HORIZONTAL VELOCITY REMAINS CONSTANT.

- It is worth noting that a particle which is fired from the top of a cliff at 100 ms^{-1} horizontally and another which is dropped from the top of the same cliff at the same time will land on the ground at the same time! This is because they both have an initial vertical component of velocity of 0 ms^{-1}. Their initial horizontal components of velocity will have no effect on their motion in the vertical direction.

Direction of flight

- The direction of flight depends upon the ratio of the horizontal and vertical velocities.
- As the horizontal velocity remains constant, the direction of flight changes because of the change in the vertical velocity.

- The direction of flight can be obtained by combining the velocity components in the usual way:



$$\tan\theta = \frac{v_y}{v_x}$$

Finding the time of flight, range and maximum height

Time of flight

The time of flight can be found in two ways:-

- Use $v = 0$ in $v = u + at$ to find the time to maximum height, and then double it. *This only works if the starting and finishing points are on the same level.*
- Put $y = h$ in $s = ut + \frac{1}{2}at^2$, where h is the vertical displacement from its starting point when the particle lands, then solve this quadratic to give t. If the projectile starts and stops at the same level, $h = 0$. There will be two solutions to the quadratic, but the time when the particle lands must be the greater *(think about why this is the case).*

Range

The range is found by multiplying the time of flight with the horizontal component of the velocity.
(*Remember, for a projectile, the horizontal component of velocity is* **CONSTANT**).
It may be stating the obvious, but the range is always increasing whilst the particle is off the ground.

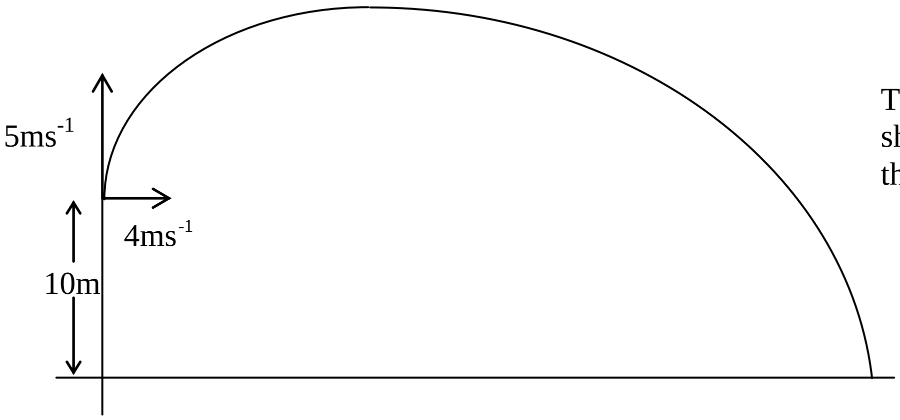
Maximum height

At the maximum height, the vertical component of velocity is 0, so use $v = 0$ in $v^2 = u^2 + 2as$ to get the maximum height and in $v = u + at$ to get the time to maximum height.

Example 1

A ball is projected with a velocity $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ from a position $\begin{pmatrix} 0 \\ 10 \end{pmatrix}$. Assuming g = 10ms^{-2} find:

(i) the maximum height
(ii) the time of flight
(iii) the range.
(iv) the angle of flight after 1 second.



The curved line shows the path of the projectile

Solution

(i) At the maximum height, the vertical velocity is 0.
So, using the equation

$$v^2 = u^2 + 2as$$
$$0^2 = 5^2 + 2(\text{-}10)s$$
$$s = \frac{25}{20} = 1.25$$

So the maximum height is 1.25m above the starting point, 11.25m above the ground.

(ii) The time of flight can be obtained using $s = ut + \frac{1}{2}at^2$, considering the vertical motion of the projectile. Here *s* will be –10 m, as we want the time when the particle hits the ground. At this time the particle is 10 m below the starting point.

$$s = ut + \tfrac{1}{2}t^2$$
$$-10 = 5t + \tfrac{1}{2}(-10)t^2$$
$$5t^2 - 5t - 10 = 0$$
$$5(t+1)(t-2) = 0$$

So $t = -1$, which is impossible, or $t = 2$.

The time of flight is 2 seconds.

(iii) The range is just the horizontal velocity, which remains constant, multiplied by the time of flight.

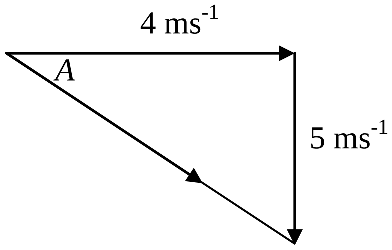
So the range is $4 \times 2 = 8$ m

(iv) The direction of the flight is given by the ratio of the velocities.

After 1 second, the vertical velocity is found using $v = u + at$

$$v = 5 + (-10) \times 1$$
$$v = -5$$

The horizontal velocity is still 4 ms^{-1}. It does not change.



Angle of flight:

$\tan A = \frac{5}{4}$

$A = 51.3^\circ$ (3s.f.)

So, after 1 second, the direction of flight is 51.3° (3 s.f.) below the horizontal.