Section 1: Resolving forces

Notes and Examples

These notes contain subsections on

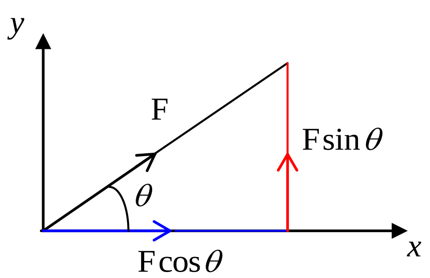
Resolving forces

You have already worked with forces in two dimensions in cases where the force is given in the form $a\mathbf{i}+b\mathbf{j}$ or $\begin{pmatrix} a \\ b \end{pmatrix}$. When forces are given in this form, it is easy to add two or more forces to find the total (resultant) force.

Often, forces in two dimensions are not conveniently given in component form. So it is often useful, when working in two dimensions, to be able to split a force up into two perpendicular components (e.g. vertically and horizontally). This is called **resolving** the force. The combined effect of these components is the same as the effect of the original force.

Resolving forces is absolutely fundamental to mechanics. Here is a summary of the method:

- Draw the force, magnitude F
- Mark on your *x* and *y* axes
- Mark on one angle, which you know
- Make your vector into a right–angled triangle.
- Imagine swinging the vector through the angle until it lies on one of the other sides of the triangle
- This component is $\text{F}\times\cos\theta$
- The other component is $\text{F}\times\sin\theta$



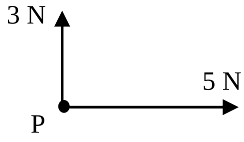
Edexcel A level Forces in 2D 1 Notes and Examples

Resultant forces

If several forces act on an object, they can be replaced by a single force which has the same effect as the combination of the individual forces. This single force is called the resultant force.

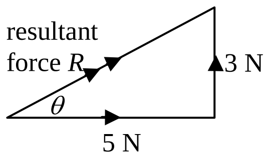
Here is a simple example showing how you can find the resultant of two forces acting in perpendicular directions.

Example 1
Two forces of 3 N and 5 N act on a particle P, as shown in the diagram below.



What is the magnitude and direction of the resultant force on the particle?

Solution
The resultant force can be found by drawing the forces 'nose to tail', forming a triangle.



The magnitude of the resultant force can be found using Pythagoras' theorem.

$$R^2 = 3^2 + 5^2 = 34$$

$$R = \sqrt{34} = 5.83$$

The direction of the resultant force can be found using trigonometry.

$$\tan\theta = \frac{3}{5}$$

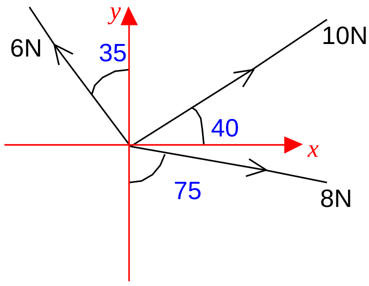
$$\theta = 31.0°$$

The resultant force has magnitude 5.83 N and acts at 31.0° to the 5 N force.

To find the resultant of forces which do not act in perpendicular components, it is usually easiest to resolve the forces into components. You can then find the sum of the components in each direction, and then combine these using Pythagoras' theorem, as in Example 1.

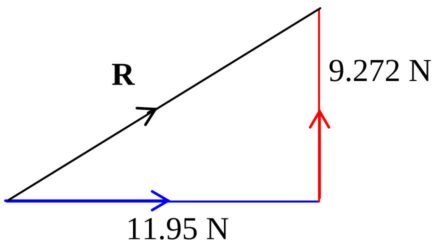
If you must resolve a number of vectors and find their resultant, it is best if you draw up a table.

Example 2
What single force can replace the following system of forces?



Solution

Force	*x* direction	*y* direction
10 N	10cos 40 = 7.660	10 sin 40 = 6.428
8 N	8sin 75 = 7.727	-8cos 75 = -2.071
6 N	-6sin 35 = -3.441	6cos 35 = 4.915
Resultant force, R	11.95	9.272



The single force that can replace the system has magnitude:

$$|\mathbf{R}| = \sqrt{11.95^2 + 9.272^2} = 15.1 \text{ N (3 s.f.)}$$

Notice workings are kept to 4 s.f. to give a final answer accurate to 3 s.f. - this avoids rounding errors

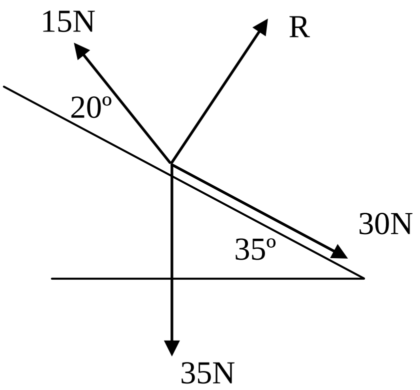
and direction:

$$\tan^{-1}\left(\frac{9.272}{11.95}\right) = 37.8° \text{ (3 s.f.) anticlockwise from the positive } x \text{ direction.}$$

Problems involving slopes

In situations involving particles on slopes, like in example 3 below, it is usually easiest to work with directions parallel and perpendicular to the slope.

Example 3
An object sliding on a slope is subjected to the forces shown in the diagram below. The resultant force acts down the slope. Resolve all the forces parallel and perpendicular to the slope to find the magnitude of the resultant force and the magnitude of the normal reaction, *R* N.



Solution

Force	Parallel to the slope	Perpendicular to the slope
15N	-15cos 20º = -14.10	15 sin 20º = 5.13
RN	0	*R*
30N	30	0
35N	35sin 35° = 20.08	-35cos 35° = -28.67
Resultant	35.98	*R* – 23.54

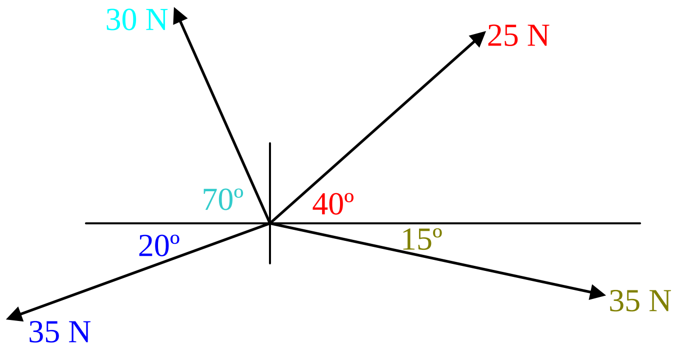
Since the resultant force acts down the slope, the resultant force in the direction perpendicular to the slope must be 0, so *R* = 23.5 N (3s.f.), so the magnitude of the normal reaction is 23.5 N.

The magnitude of the resultant force is 36.0 N (3s.f.)

Forces in equilibrium

If a system of forces is in equilibrium, then the resultant force must be zero. A particle which is at rest is in equilibrium.

Example 4
Resolve each force below into a horizontal and vertical component and decide whether the system is in equilibrium.



Solution

Force	Horizontal component (**i**)	Vertical component (**j**)
25 N	25cos 40 = 19.15	25sin 40 = 16.07
35 N	35cos 15 = 33.81	-35sin 15 =-9.06
35 N	-35cos20 = -32.89	-35sin 20 = -11.97
30 N	-30cos 70 =-10.26	30 sin 70 = 28.19
Resultant (total of each component)	9.81	23.23

Be careful to get the signs right.
A table, like this one, can help
you to avoid mistakes.

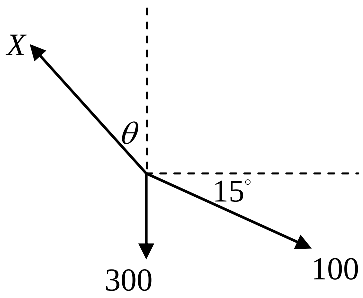
So there is a resultant force of 9.81 N in the horizontal direction and a resultant force of 23.2 N in the vertical direction, and the system is therefore not in equilibrium (if it were in equilibrium, both components would have to be 0).

The easiest way to consider the equilibrium of a particle is to resolve all forces acting on the particle into two perpendicular components (usually horizontal and vertical, or in the case of a slope, parallel to the slope and perpendicular to the slope). Since the total of the components in each direction must be zero, this will give you two equations. So you can only solve equilibrium problems involving two unknowns: these might be the magnitudes of forces, or they could be angles.

In this work you are dealing only with coplanar forces (forces that act in the same plane – most often a vertical plane, but the plane could be in any direction). If you were working with forces which were not coplanar, then you would need to resolve forces into three perpendicular directions (such as the x, y and z directions) and you would then be dealing with three equations and there could be up to three unknowns.

Example 5 below deals with a problem in which the unknowns are one force and one angle.

Example 5
The diagram below shows three forces acting on a particle which is in equilibrium. Find the values of the unknown force X and the angle θ.



Solution

Considering the horizontal components: $100\cos 15° - X\sin\theta = 0$

$X\sin\theta = 100\cos 15° \quad (1)$

Considering the vertical components: $X\cos\theta - 100\sin 15° - 300 = 0 \quad (2)$

$X\cos\theta = 100\sin 15° + 300$

Dividiing (1) by (2): $\tan\theta = \dfrac{100\cos 15°}{100\sin 15° + 300}$

$\theta = 16.5°$ (1 d.p.)

Substituting into (1): $X = \dfrac{100\cos 15°}{\sin 16.51°} = 340$ (3 s.f.)