Section 1: Motion in two dimensions

Section test

1. A particle, initially moving with velocity 5**i** – 2**j** ms^{-1} is subject to a constant acceleration of 2**i** + **j** ms^{-2}. What is the velocity of the particle at t = 5?

2. A particle which is initially moving with a velocity of -4**i** + 3**j** ms^{-1} is subject to a constant acceleration. After 3 seconds its new velocity is 2**i** + **j** ms^{-1}. What is the displacement of the particle?

3. A particle has velocity $\mathbf{v} = \left(t^2\mathbf{i} + 3t\mathbf{j}\right)\text{ms}^{-1}$ at time t.
   Find the acceleration of the particle when t = 2 .

4. The velocity of a particle is modelled by $\mathbf{v} = \left(3t^2\mathbf{i} + 10t\mathbf{j}\right)\text{ms}^{-1}$. Given that $\mathbf{r} = \left(4\mathbf{i} - 4\mathbf{j}\right)\text{m}$ when t = 0, find the distance of the particle from the origin when t = 2.

5. If a particle has acceleration $\mathbf{a} = \left((4t+5)\mathbf{i} + 6t\mathbf{j}\right)\text{ms}^{-2}$ and the particle is initially at rest, find its velocity when t = 4.

6. A bird flies in a vertical plane from a point on the ground. Its position from the time when it leaves the ground until it returns to the ground again is modelled as $\mathbf{r} = \left(\frac{1}{4}\left(t^2+2t\right)\mathbf{i} + \frac{1}{10}\left(-t^3+t^2+6t\right)\mathbf{j}\right)\text{m}$, where **i** is a unit vector in the horizontal direction and **j** is a unit vector in the vertical direction.
   Find the time at which it reaches the ground again
   Find its velocity as it hits the ground.

7. The position of a racing car can be modelled as $\mathbf{r} = \left(\left(t^2-5t\right)\mathbf{i} + \frac{1}{5}\left(t^3+5t^2-7t\right)\mathbf{j}\right)\text{m}$ for a 4 second period during a race. The starting point of the model is taken as t = 0.
   Find the velocity of the car after 3 seconds.
   Find the acceleration of the car after 1 second.

8. The acceleration of a particle is given by $\left(\left(t^2+2\right)\mathbf{i} + 2t\mathbf{j}\right)\text{ms}^{-2}$. Its initial velocity is $\left(6\mathbf{i} + 7\mathbf{j}\right)\text{ms}^{-1}$ and it starts initially at the point (5**i** + 6**j**) m. Find its position after 6 seconds.

Edexcel A level Maths Kinematics 1 section test

Solutions to section test

1. $\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t$

$\underset{\sim}{v} = 5\underset{\sim}{i} - 2\underset{\sim}{j} + 5(2\underset{\sim}{i} + \underset{\sim}{j})$

$\underset{\sim}{v} = 15\underset{\sim}{i} + 3\underset{\sim}{j}$

2. $\underset{\sim}{r} = \frac{1}{2}(\underset{\sim}{u} + \underset{\sim}{v})t$

$= \frac{1}{2}\left(-4\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{i} + \underset{\sim}{j}\right) \times 3$

$= \frac{3}{2}\left(-2\underset{\sim}{i} + 4\underset{\sim}{j}\right)$

$= -3\underset{\sim}{i} + 6\underset{\sim}{j}$

3. $\underset{\sim}{v} = t^2\underset{\sim}{i} + 3t\underset{\sim}{j}$

$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = 2t\underset{\sim}{i} + 3\underset{\sim}{j}$

When $t = 2$, $\underset{\sim}{a} = 2 \times 2\underset{\sim}{i} + 3\underset{\sim}{j} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$

The acceleration when $t = 2$ is $\left(4\underset{\sim}{i} + 3\underset{\sim}{j}\right)$ ms^{-2}.

4. $\underset{\sim}{r} = \int \underset{\sim}{v}\, dt = \int \left(3t^2\underset{\sim}{i} + 10t\underset{\sim}{j}\right) dt = t^3\underset{\sim}{i} + 5t^2\underset{\sim}{j} + \underset{\sim}{c}$

When $t = 0$, $\underset{\sim}{r} = 4\underset{\sim}{i} - 4\underset{\sim}{j} \Rightarrow \underset{\sim}{c} = 4\underset{\sim}{i} - 4\underset{\sim}{j}$,

so $\underset{\sim}{r} = t^3\underset{\sim}{i} + 5t^2\underset{\sim}{j} + 4\underset{\sim}{i} - 4\underset{\sim}{j}$

When $t = 2$, $\underset{\sim}{r} = 2^3\underset{\sim}{i} + 5 \times 2^2\underset{\sim}{j} + 4\underset{\sim}{i} - 4\underset{\sim}{j} = 12\underset{\sim}{i} + 16\underset{\sim}{j}$

Distance of particle from origin $= \sqrt{12^2 + 16^2} = 20$ m

5. $\underset{\sim}{v} = \int \underset{\sim}{a}\, dt = \int \left((4t + 5)\underset{\sim}{i} + 6t\underset{\sim}{j}\right) dt = (2t^2 + 5t)\underset{\sim}{i} + 3t^2\underset{\sim}{j} + \underset{\sim}{c}$

When $t = 0$, $\underset{\sim}{v} = 0 \Rightarrow \underset{\sim}{c} = 0$

so $\underset{\sim}{v} = (2t^2 + 5t)\underset{\sim}{i} + 3t^2\underset{\sim}{j}$

When $t = 4$, $\underset{\sim}{v} = (2 \times 4^2 + 5 \times 4)\underset{\sim}{i} + 3 \times 4^2\underset{\sim}{j} = 52\underset{\sim}{i} + 48\underset{\sim}{j}$

The velocity of the particle is $\left(52\underset{\sim}{i} + 48\underset{\sim}{j}\right)$ ms^{-1}.

6. It reaches the ground when the component of $\underset{\sim}{r}$ in the $\underset{\sim}{j}$ direction is zero.

$$-t^3 + t^2 + 6t = 0$$
$$t(t^2 - t - 6) = 0$$
$$t(t-3)(t+2)$$
$$t = 0,\ 3,\ -2$$

Since t must be positive, it reaches the ground after 3 seconds..

$$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = \tfrac{1}{4}(2t+2)\underset{\sim}{i} + \tfrac{1}{10}\left(-3t^2 + 2t + 6\right)\underset{\sim}{j}$$

When $t = 3$, $\underset{\sim}{v} = \tfrac{1}{4}(2\times3+2)\underset{\sim}{i} + \tfrac{1}{10}\left(-3\times3^2 + 2\times3 + 6\right)\underset{\sim}{j} = 2\underset{\sim}{i} - 1.5\underset{\sim}{j}$

Its velocity as it hits the ground is $\left(2\underset{\sim}{i} - 1.5\underset{\sim}{j}\right)$ ms^{-1}.

7. $\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = (2t-5)\underset{\sim}{i} + \tfrac{1}{5}\left(3t^2 + 10t - 7\right)\underset{\sim}{j}$

When t = 3, $\underset{\sim}{v} = (2\times3-5)\underset{\sim}{i} + \tfrac{1}{5}\left(3\times3^2 + 10\times3 - 7\right)\underset{\sim}{j} = \underset{\sim}{i} + 10\underset{\sim}{j}$

The velocity of the car after 3 seconds is $\left(\underset{\sim}{i} + 10\underset{\sim}{j}\right)$ ms^{-1}.

$$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = 2\underset{\sim}{i} + \tfrac{1}{5}(6t+10)\underset{\sim}{j}$$

When t = 1, $\underset{\sim}{a} = 2\underset{\sim}{i} + \tfrac{1}{5}(6+10)\underset{\sim}{j} = 2\underset{\sim}{i} + 3.2\underset{\sim}{j}$

The acceleration of the car after 1 second is $\left(2\underset{\sim}{i} + 3.2\underset{\sim}{j}\right)$ ms^{-2}.

8. $\underset{\sim}{v} = \int \underset{\sim}{a}\,dt = \int\left(\left(t^2+2\right)\underset{\sim}{i} + 2t\underset{\sim}{j}\right)dt = \left(\tfrac{1}{3}t^3 + 2t\right)\underset{\sim}{i} + t^2\underset{\sim}{j} + \underset{\sim}{c}$

When t = 0, $\underset{\sim}{v} = 6\underset{\sim}{i} + 7\underset{\sim}{j}$, so $\underset{\sim}{c} = 6\underset{\sim}{i} + 7\underset{\sim}{j}$.

$$\underset{\sim}{v} = \left(\tfrac{1}{3}t^3 + 2t\right)\underset{\sim}{i} + t^2\underset{\sim}{j} + 6\underset{\sim}{i} + 7\underset{\sim}{j} = \left(\tfrac{1}{3}t^3 + 2t + 6\right)\underset{\sim}{i} + \left(t^2 + 7\right)\underset{\sim}{j}$$

$$\underset{\sim}{r} = \int \underset{\sim}{v}\,dt = \left(\tfrac{1}{12}t^4 + t^2 + 6t\right)\underset{\sim}{i} + \left(\tfrac{1}{3}t^3 + 7t\right)\underset{\sim}{j} + \underset{\sim}{d}$$

When t = 0, $\underset{\sim}{r} = 5\underset{\sim}{i} + 6\underset{\sim}{j}$, so $\underset{\sim}{d} = 5\underset{\sim}{i} + 6\underset{\sim}{j}$

$$\underset{\sim}{r} = \left(\tfrac{1}{12}t^4 + t^2 + 6t\right)\underset{\sim}{i} + \left(\tfrac{1}{3}t^3 + 7t\right)\underset{\sim}{j} + 5\underset{\sim}{i} + 6\underset{\sim}{j}$$
$$= \left(\tfrac{1}{12}t^4 + t^2 + 6t + 5\right)\underset{\sim}{i} + \left(\tfrac{1}{3}t^3 + 7t + 6\right)\underset{\sim}{j}$$

When t = 6,

$$\underset{\sim}{r} = \left(\tfrac{1}{12}\times6^4 + 6^2 + 6\times6 + 5\right)\underset{\sim}{i} + \left(\tfrac{1}{3}\times6^3 + 7\times6 + 6\right)\underset{\sim}{j} = 185\underset{\sim}{i} + 120\underset{\sim}{j}$$

Its position is $\left(185\underset{\sim}{i} + 120\underset{\sim}{j}\right)$ m