Section 1: The moment of a force

Notes and Examples

These notes contain subsections on

- **Rigid bodies**
- **Moments and equilibrium**

Rigid bodies

In all the work you have done so far in Mechanics, you have modelled objects as particles. A particle has no size, so all forces acting on it act through its centre of mass. However, if you consider a larger object such as a plank, with forces acting on it at different points, the forces have a turning effect, so it is not reasonable to consider the object as a particle.

Note that even quite a large body can be reasonably considered as a particle if all the forces on it act through the same point. This explains why you were able to use the particle model in previous work in Mechanics, even for large objects such as a car!

All the objects you deal with in this chapter are considered to be rigid bodies: i.e. they do not change shape when forces are applied to them. This is a reasonable assumption in most circumstances, although in reality many planks bend when a large weight is placed on them.

Moments and equilibrium

The moment of a force about a point O is given by

Moment = force $\times$ perpendicular distance of the force from O.

So a larger force produces a larger moment, but also a force acting further from the point produces a larger moment. This principle is used in real life when using a lever to lift a heavy object – a longer lever is more effective.

The moment of a force about a point produces a turning effect about the point. So for a rigid body to be in equilibrium, not only must there be no resultant force acting on the body, but there must be no net turning effect. So the total moment about any given point must be zero.
Where there is a hinge or fulcrum there is always some kind of reaction force at the hinge or fulcrum. This is why it often makes sense to take moments about the hinge or fulcrum, as the reaction force has no moment about that point.

Many problems can be solved by using a combination of resolving forces and taking moments. In A level Mathematics the problems you meet will be restricted to those in which all the forces act in parallel directions, so only one equation can be obtained by resolving forces. A second equation is obtained by taking moments about a suitable point. Alternatively, you can solve problems by taking moments about two different points, and not resolving forces at all. Either method is equally valid, and the choice in this case is really down to personal preference.

Example 1

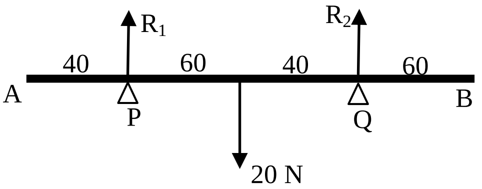
A uniform plank AB of length 2 m and weight 20 N rests on two supports, one 40 cm from A and the other 60 cm from B.

(i) Find the reaction force at each support.

(ii) A cat of mass 40 N starts from the centre of the plank and walks towards A. How far from A is the cat when the plank tips?

Solution

(i)



Taking moments about P: $100R_2 = 60 \times 20 = 0$

$R_2 = 12$

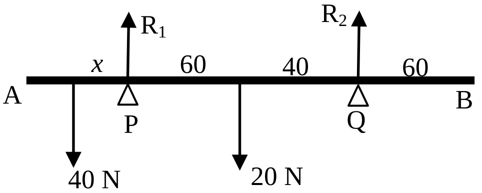
Resolving vertically: $R_1 + R_2 - 20 = 0$

$R_1 + 12 - 20 = 0$

$R_1 = 8$

The reaction forces at the supports are 12 N and 8 N.

(ii)



When the plank starts to tip, the reaction force at Q is zero.

Taking moments about P: $60 \times 20 - 40x = 0$

$x = 30$

So the cat is 10 cm from A when the plank tips.

Note that you cannot solve problems involving three unknowns in which all the forces act in parallel directions. If you write down an equation by resolving forces and two equations by taking moments about two different points, you will find that each of the equations could be obtained by combining the other two in some way. So the problems that you meet will be restricted to two unknowns.

If you go on to study Mechanics in Further Mathematics, you will look at situations in which the forces do not all act in parallel directions, such as a ladder leaning against a wall. In such cases you can solve a problem involving three unknowns by finding three equations: either resolving forces in two directions and taking moments about one point, or by resolving forces in one direction and taking moments about two points.