## Edexcel A level Maths Moments

## Section 2: Moments of forces at an angle

## Notes and Examples

These notes contain subsections on

- The moment of a force which acts at an angle
- Solving equilibrium problems using moments


## The moment of a force which acts at angle

In A level Mathematics the idea of the moment of a force was introduced. The moment of a force is found by multiplying the force by the perpendicular distance from the point about which the moment is taken.

In all the problems in A level Mathematics involving moments, forces acted in a direction perpendicular to the line between the fulcrum and the point at which the force acted. This made it easy to calculate the moment of the force.


In the diagram above, the force $F$ is perpendicular to OA, so the moment of the force about O is given by Fd .

In the diagram below, however, the force F is not perpendicular to OA .


There are two ways to think about finding the moment of this force (both giving the same result).

The first approach involves finding the perpendicular distance between the line of action of the force and the point O .

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The diagram above shows that the perpendicular distance between the line of action of the force and the point O is $d \sin \theta$. Therefore the moment of the force is $F d \sin \theta$.

The alternative approach is to resolve the force $F$ into components, one component parallel to OA and the other perpendicular to OA, as shown below.


The component parallel to $\mathrm{OA}, F \cos \theta$, goes through O and therefore its moment about $O$ is zero.
The component perpendicular to $\mathrm{OA}, F \sin \theta$, acts at a perpendicular distance of $d$ from O , and so its moment about O is $F \sin \theta \times d$, or $F d \sin \theta$.

Both approaches to finding the moment are equivalent. The one you choose to use really depends on which one you find easier to visualise.

## Solving problems using moments

In work on equilibrium in A level Mathematics, you have solved two-dimensional problems involving two unknown forces. You do this by resolving in two perpendicular directions to obtain two equations describing the equilibrium. The directions involved are often horizontal and vertical, but you have also met situations, such as a particle on a slope, where it is easier to resolve in different directions, such as parallel to the slope and perpendicular to the slope.

When solving a problem involving a rigid body, you must also consider possible turning motion, as well as possible movement in two dimensions. This means that you can find three equations to describe the equilibrium, and therefore you can solve problems involving three unknown quantities. For example, you can find three unknown forces by resolving horizontally and vertically and taking moments about one point. Alternatively you could solve the same problem by resolving in one direction and taking moments about two points.

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Note, however, that even though you could write down four equations describing the equilibrium, by resolving in two directions and taking moments about two points, you cannot solve a problem involving four unknown forces. You would find that any one of your four equations could be obtained by combining the other three.

When you are making a decision about how to solve a problem, remember that it is generally easier to take moments about a point which has the lines of action of several forces going through it.

Where there is a hinge or fulcrum in a problem, there is always some kind of reaction force at the hinge or fulcrum. This is why it often makes sense to take moments about the hinge or fulcrum, as the reaction force has no moment about that point.

Here is an example.


Example 1
A uniform plank AB of mass 25 kg is pivoted at A and held at an angle of $30^{\circ}$ to the vertical by a force applied at B , perpendicular to AB . Find this force.

Solution


Let the length of the plank be $2 x$.
Taking moments about A (as both $R_{x}$ and $R_{y}$ go through that point)

$$
\begin{aligned}
& 25 g x \cos 60^{\circ}=F \times 2 x \\
& F=\frac{25 g \cos 60^{\circ}}{2} \\
& F=61.25 \mathrm{~N}
\end{aligned}
$$

Ladders are a particular example of a statics problem. For a ladder to lean against a wall in equilibrium, the ground must be rough, so that there is a horizontal frictional force to counteract the horizontal reaction force at the top of the ladder where it is in contact with the wall.


## Example 2

A uniform ladder of length 12.5 m and mass 48 kg rests with its top against a smooth wall and its foot on rough ground, 3.5 m from the base of the wall. Find the frictional and reaction forces at the base of the ladder.

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Solution


Resolving vertically:

$$
R_{g}=48 \mathrm{~g}=48 \times 9.8=470.4
$$

The reaction force at the ground is 470.4 N
Resolving horizontally:

$$
R_{w}=F
$$

Taking moments about the base of the ladder:

$$
\begin{aligned}
& 12.5 R_{w} \sin \alpha-6.25 \times 48 g \cos \alpha=0 \\
& 12.5 F \times \frac{12}{12.5}=6.25 \times 48 \times 9.8 \times \frac{3.5}{12.5}
\end{aligned}
$$

$$
F=68.6 \mathrm{~N}
$$

The frictional force at the base of the ladder is 68.6 N

