Section 1: Friction

Notes and Examples

These notes contain subsections on

- Friction
- The direction of the frictional force

Friction

So far, surfaces that you have encountered have been described as smooth. This means that there is no friction between the object and the surface. In this section you will consider situations involving rough surfaces, so that there is friction between the object and the surface, which opposes any tendency to motion.

Obviously, some surfaces are rougher than others, and will therefore produce greater frictional forces. The coefficient of friction for a surface, usually denoted by μ, gives a measure of the roughness of the surface (a smooth surface has coefficient of friction zero).

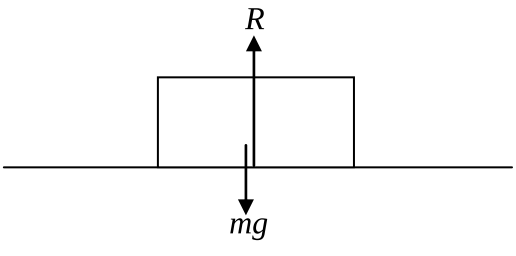
A heavier object also results in a greater frictional force. In fact, the frictional force depends not on the weight of the object but on the normal reaction force, R, between the object and the surface.

The frictional force F is given by $F \leq \mu R$. The special case $F = \mu R$ can only be used when a particle is about to move, or is moving – when it is in *limiting equilibrium*.

If the object is not in limiting equilibrium, then $F < \mu R$.

It is essential to realise that the size and the direction of the frictional force between two surfaces varies according to circumstances.

For example, consider a block of mass m kg resting on a rough horizontal plane.



The only forces acting on the block are its weight and the reaction force. Even though the surface is rough, there is no frictional force.

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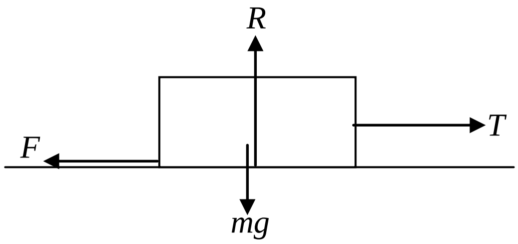
If a small horizontal force T is applied to the block, not enough to make it slide, a frictional force F opposes the motion.



In this case, as the block is not moving, F must be equal to T.

If T is increased, F will also increase. However, eventually T will become large enough to move the block. In this case F has reached its maximum value of μR.

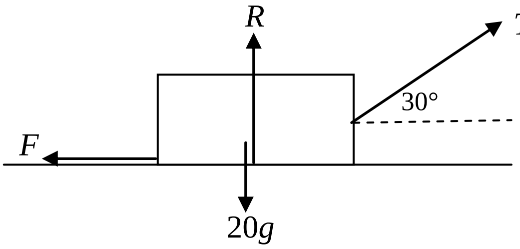
Once T exceeds the maximum value of F, there will be a net force in the direction of T, which causes the block to move.

You will learn later in this course that if a resultant force acts on a particle, then the particle will accelerate. If the resultant force is zero, then the acceleration is zero, which may mean that the particle is at rest, but it could also mean that the particle is moving at constant speed.

Therefore when T is equal to μR (the maximum value of F), there are two possibilities: the block may be stationary but on the point of moving, or it may be moving at constant speed (so that its acceleration is zero.) However, for the block to reach a constant speed it must first accelerate from an initial speed of zero to the required speed. To move the block at constant speed, therefore, T needs to be greater than μR initially, so that the block accelerates, after which T can be reduced to a value equal to μR. This shows, as you know from practical experience, that it is harder to start an object moving than it is to keep it moving once it has started!

Example 1
A box of mass 20 kg is to be pulled across a rough floor by a string inclined at 30° to the horizontal. The coefficient of friction between the box and the floor is 0.6. Find the tension in the string when the box is on the point of moving.

Solution



Resolving vertically: $R + T\sin 30 - 20g = 0$

$R = 196 - T\sin 30°$

Friction is limiting, so $F = \mu R$

$= 0.6(196 - T\sin 30°)$

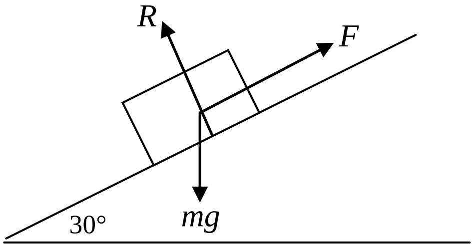
Resolving horizontally:

$$T\cos 30° - F = 0$$
$$T\cos 30° = 0.6(196 - T\sin 30°)$$
$$T\cos 30° + 0.6T\sin 30° = 0.6 \times 196$$
$$T(\cos 30° + 0.6\sin 30°) = 117.6$$
$$T = 100.9$$

The tension in the string is 100.9 N.

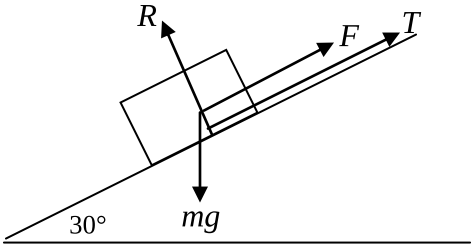
The direction of the frictional force

The direction of the frictional force also varies according to circumstances. Consider a block of mass m kg at rest on a rough plane inclined at 30° to the horizontal.



Here the frictional force F acts to prevent the block from sliding down the plane.

If a small force T is applied up the slope, then the frictional force becomes smaller, as the force T is helping to stop the block sliding down the plane. If T is increased, eventually no frictional force will be needed to stop the block from sliding down the plane.



If T is increased further, the tendency of the block will no longer be to slide down the plane, but to slide up. This means that the frictional force F will now act in the opposite direction, to oppose the tendency to slide up the plane.

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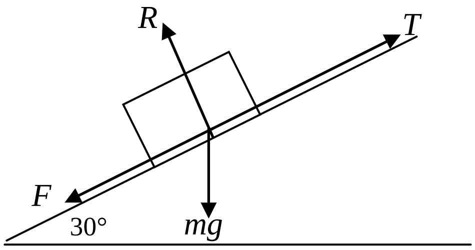

As T continues to increase, the value of F increases until it reaches its maximum value of μR. Any further increase of T after this point will result in the block accelerating up the plane.

In most questions that you meet, you will be told that the particle is about to slip, or that it is in limiting equilibrium. This tells you that you can use the relationship $F = \mu R$. Together with the equations formed by resolving in two directions, this gives you three equations, so that you can now solve problems involving three unknowns (one of which might be the coefficient of friction).

