Section 1: Vectors in three dimensions

Exercise level 2

1. Using vector methods show that X(3, -4, 0), Y(-1, 8, -8) and Z(6, -13, 6) are collinear.

2. A, B, and C have position vectors $-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $8\mathbf{i} + 7\mathbf{j} - 9\mathbf{k}$ and $2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ respectively. Prove that triangle ABC is right angled and find its area.

3. Three vectors are $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 4 \\ 1 \\ k \end{pmatrix}$.
   Given that $p\mathbf{a} + q\mathbf{b} = \mathbf{c}$, find the values of p, q and k.

4. Three points have coordinates E (1, 2, -5), F (0, 4, 2) and G (3, 1, -1). Find the point H such that EFGH forms a parallelogram.

5. A vector $\mathbf{p}$ is parallel to the vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, and $|\mathbf{p}| = 24$. Find $\mathbf{p}$.