

## Section 2: Numerical integration

### Exercise level 3 (Extension)

- [You need to use radians in the trig equation throughout this question. Make sure your calculator is set to 'radian mode'.]  
A scientist is estimating the mass of water in a wave at sea. The wave has a height from trough to peak of 5 metres, and the wave extends approximately at right-angles for 175 metres. The scientist models the cross-section of the wave by the graph  
$$y = a + b \cos x \text{ for } -\pi \leq x \leq +\pi$$
with the trough of the wave given by  $y = 0$ .
  - Suggest values for  $a$  and  $b$ .
  - Sketch the graph of the cross-section.
  - Use the trapezium rule with 6 strips to find the area of the cross-section between  $x = 0$  and  $\pi$ .
  - If one cubic metre of sea water has a mass of approximately 1 tonne, estimate the total mass of water in the wave.
- Find  $I = \int_0^1 \sqrt[4]{x} \, dx$ , giving your answer to 4 decimal places.
  - Use the trapezium rule with 4 strips to estimate  $I$  to 4 decimal places.
  - Find the percentage error in the estimate.
  - With the help of a sketch, explain why the trapezium rule gives a poor estimate in this example.