Section 2: Numerical integration

Exercise level 2

1. Use the trapezium rule with 3 strips to find an approximate value for the integral $\int_0^{\pi/2} \frac{1}{\sqrt{(1+\cos\theta)}}\,\mathrm{d}\theta$.

2. Use the trapezium rule with
   (i) 3 ordinates
   (ii) 5 ordinates
   to find an approximate value for $\int_0^1 \mathrm{e}^{-x^2}\,\mathrm{d}x$.

   Find the second derivative of e^{-x^2} and explain how this shows that it is not possible to say whether the estimates are overestimates or underestimates.

3. (i) Use calculus to prove that the trapezium rule will always give an overestimate for the value of the integral $\int_0^1 \mathrm{e}^{-\sin x}\mathrm{d}x$.

   (ii) Find underestimates and overestimates for the value of $\int_0^1 \mathrm{e}^{-\sin x}\mathrm{d}x$
   (a) using rectangles of width 0.25,
   (b) using rectangles of width 0.1.

4. (i) Using the trapezium rule with 5 strips, find an approximate value to 4 d.p. for $\int_0^{0.5} (1+x^2)^{12}\,\mathrm{d}x$.
   (ii) Using the binomial expansion expand $(1 + x^2)^{12}$ in ascending powers of x up to and including the term in x^8.
   (iii) Use the expansion in (ii) to obtain an approximation to 4 d.p. for the integral in (i).
   (iv) Explain how you know that the estimate in (i) is an overestimate and that in (iii) an underestimate. What could you do to improve the accuracy of both methods and bring the answers closer together?

5. (i) Use the trapezium rule to find an estimate for the value of $\int_0^1 \ln(x^2+1)\,\mathrm{d}x$ using strips of width 0.2.
   (ii) Use calculus to determine whether this estimate is an overestimate or an underestimate.
   (iii) Use rectangles of width 0.2 to find another bound for the value of the integral.