

## Section 2: Integration by substitution

## Exercise level 2

- Find the area enclosed by the curve  $y = \sqrt{1+2x}$ , the  $x$ -axis and the line  $x = 4$ .
- Find by integrating by inspection  $\int \frac{x}{\sqrt{1+x^2}} dx$ . Verify your result using the substitution  $u = \sqrt{1+x^2}$ .
- Find (i)  $\int \frac{1}{2x+1} dx$  (ii)  $\int \frac{3x}{1+x^2} dx$
- Find
  - $\int \cot^2 2x dx$
  - $\int \frac{\tan x}{\sin 2x} dx$
  - $\int \sin^2 3x dx$
  - $\int (\sin 2x + \cos 2x)^2 dx$
- Find
  - $\int_0^{\frac{\pi}{4}} \sin^3 2x dx$
  - $\int_0^{\frac{\pi}{2}} \cos^4 x dx$
  - $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \tan \frac{1}{2} x dx$
  - $\int_0^{\pi/2} (1 + \cos x)^3 dx$
- Evaluate  $\int_0^1 \frac{x^2}{x^3+2} dx$ , expressing your answer as a single logarithm.
- (i) Find  $\int \frac{1-2x}{1+x-x^2} dx$   
 (ii) Find  $\int e^x(1+e^x) dx$ .
- Differentiate  $x \ln x$ . Use this result to find the integral of  $\ln x$ .
- Use integration by inspection to find
  - $\int \frac{\cos x}{1+2 \sin x} dx$
  - $\int_0^1 \frac{1+e^x}{x+e^x} dx$
- Find  $\int \cos^2 x \sin x dx$ .
- Find the area bounded by the curve  $y = 2^x$ , the  $x$ -axis, the  $y$ -axis and the line  $x = 1$ . Give your answer in terms of  $\ln 2$ .