Section 1: Solving equations numerically

Exercise level 2

1. (i) By considering turning points, show that the equation $x^4 + 4x^3 + 5x^2 - 9 = 0$ has only one positive root.

    (ii) Show that the equation can be rearranged to the form $x = \sqrt{\dfrac{9}{x^2 + 4x + 5}}$.

    (iii) Using the equation from (ii) as the iterative formula, find this positive root to 2 d.p.

2. (i) Show that $x^3 - x - 4 = 0$ has one real root only and find two integers between which the root lies.

    (ii) Using an initial value of $x_0 = 2$, find the values of x_1 and x_2 using each of the following iterative formulae, and draw a diagram to show how each procedure diverges or converges.

    (a) $x_{n+1} = x_n^3 - 4$

    (b) $x_{n+1} = \dfrac{x_n + 4}{x_n^2}$

    (c) $x_{n+1} = (x_n + 4)^{1/3}$

3. For the roots of the equations in (i) to (v)

    (a) find the two consecutive integers between which the root lies,

    (b) use the Newton-Raphson method to find the root correct to three decimal places.

    (i) $x^3 - 2x - 2 = 0$

    (ii) $x^3 - x + 3 = 0$

    (iii) $2x^5 - x^2 + 1 = 0$

    (iv) $x^5 - x - 1 = 0$

    (v) $3x^3 + 2x^2 - 3 = 0$