Section 1: The shape of curves

Exercise level 3 (Extension)

1. A graph has equation $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$.
   (i) Prove that the graph has a single point of inflection in all cases, and find its coordinates.
   (ii) Prove that the point of inflection is a stationary point if and only if $b^2 = 3ac$.

2. A graph has equation $y = ax^4 + bx^3 + cx^2 + dx + e$, where $a \neq 0$.
   Prove that the graph has no points of inflection if $3b^2 \leq 8ac$.

3. A quartic graph has a stationary point of inflection at (1, 37) and a non-stationary point of inflection at (-2, -125). Find the equation of the graph.