# Section 1: The shape of curves

## Exercise level 2

1. (i) Find the stationary points on the curve $y = x^4 - 2x^3$ and distinguish between them, showing all of the relevant working clearly.
   (ii) Find the non-stationary point of inflection.
   (iii) Sketch the curve.

2. A graph has equation $y = 3x^4 - 16x^3 + 30x^2 - 24x + 12$ .
   (i) Find an expression for $\frac{dy}{dx}$ .
   (ii) Factorise your expression for $\frac{dy}{dx}$, and hence show that the graph of the equation has just two points where $\frac{dy}{dx} = 0$ .
   (iii) By considering the value of the gradient on both sides of the points you found in (ii) above, show that just one of them is a turning point, and determine whether it is a maximum or minimum. What happens at the other point you found?
   (iv) Make a rough sketch of the curve.

3. The equation of a curve is $y = (x+1)(x-3)^3$.
   (i) Write the equation of the curve in the form $y = ax^4 + bx^3 + cx^2 + dx + e$.
   (ii) Find the coordinates of the points where $\frac{dy}{dx} = 0$.
   (iii) Classify the stationary points.
   (iv) Sketch the curve.

4. (i) Find the stationary points on the curve $y = \frac{1}{x} - x^2 + 3x$ and identify their nature.
   (ii) Explain how you know that there are no non-stationary points of inflection on the curve.

5. The curve $y = x^3 + px^2 + qx + r$ has a stationary point of inflection at (-1, 3), Find the coordinates of $p$, $q$ and $r$.