Section 3: Implicit differentiation

Exercise level 3

1. The normal to the curve $x^2 + xy + 2y^2 = 8$ at the point (a, b) has gradient 4. Find the possible values of a and b.

2. (i) Use the product rule to find the gradient of the curve $y = xe^x$ at the point $x = -1$.
   (ii) By considering $\ln y$, obtain the same result using implicit differentiation.

3. (i) Given $y = \tan x$, use the quotient rule to show that $\frac{dy}{dx} = 1 + \tan^2 x$.
   (ii) Given instead that $\tan x + \tan y = 4$, find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{4}$.