

Section 2: Parametric differentiation

Exercise level 3

1. The curve C has parametric equations

$$x = 8t - 2t^3, \quad y = 9 - t^2.$$

Show that the normal to C at (6, 8) intersects C at exactly two other points and find the values of t at those points.

2. The curve with parametric equations $x = \cos t + \cos 2t$ and $y = \sin t + \sin 2t$, $0 \leq t \leq 2\pi$, crosses the x -axis at three distinct points. At one of those points, the curve also crosses itself. Find all three points and identify the point where the curve crosses itself, justifying your answer. Find the equations of the two tangents at that point.
3. A curve C is defined by the parametric equations $x = t^2$, $y = t^3 - 3t$.
- Show that C has two tangents at (3, 0) and find their Cartesian equations.
 - Find the points on C where the tangent is either horizontal or vertical. Find all the points of intersection between those tangents and C.
 - Show that C is symmetric in the x -axis.
 - Sketch the curve C.